The uncertainty budget of Mixed-Numerical-Experimental-Techniques for the identification of elastic material properties from resonant frequencies.

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Abstract

The elastic properties of homogeneous, linear-elastic materials can be identified using resonant vibration analysis. Several sources of uncertainty contribute to the combined uncertainty of the measured values. This paper presents a method to handle uncertainty budgets in vibration based mixed numerical-experimental identification techniques. The presented method is evaluated with two numerical test cases. The first example considers an isotropic material, and allows to compare the presented method with the method proposed in the UNCERT Code of Practice 13 [1]. The second example considers the uncertainty budget of the identification of a coated steel plate.

1 Introduction

Resonant vibration analysis is commonly used for the identification of the elastic properties of homogeneous, linear-elastic materials. Modern laboratory practice demands the specification of the uncertainty associated with a measured value, in accordance with the ISO-Guide for the Uncertainty of Measurement (GUM) [2] . GUM-compliant uncertainty budgets identify the individual uncertainty contributions to the combined uncertainty of the measured value. A GUM-compliant uncertainty budget has been proposed for the measurement of the elastic modulus using the flexural resonance frequency of a long beam [1]. More advanced identification techniques have enabled the characterization of the elastic properties of non-homogeneous (e.g. coated substrates) or non-isotropic materials (e.g. composites). These Mixed Numerical-Experimental Techniques (MNETs) go beyond the analytical approach and require numerical techniques as they interpret the measured resonance frequencies. In this paper, the authors propose a method to estimate the uncertainty associated with the elastic properties identified with such a MNET.

2 Vibration based material identification

2.1 Standardized methods

A well-established method to determine the stiffness of a homogeneous material consists of measuring the dimensions, weight and the resonant frequencies of a carefully machined rectangular beam or rod. The ASTM E 1876 and ENV 843-2 standard procedures specify the validated analytical equations relating these measurements with the Young's modulus and shear modulus of the material. The elastic modulus can be obtained from the fundamental flexural frequency (f_f) with:

$$E = 0.9465\,\frac{m\,f_f^2\,l^3}{w\,t^3}T_1 \tag{1}$$

in which l, w, t and m are the sample's length, width, thickness and mass, respectively. T_1 is a transverse shear correction factor which depends on Poisson's ratio (ν) and thickness to length ratio.

$$T_1 = 1 + 6.585\left(1 + 0.0752\nu + 0.8109\nu^2\right)\left(\frac{t}{l}\right)^2 - 0.868\left(\frac{t}{l}\right)^4 \tag{2}$$

$$-\left(\frac{8.340\left(1 + 0.2023\nu + 2.173\nu^2\right)\left(\frac{t}{l}\right)^4}{1.000 + 6.338\left(1 + 0.1408\nu + 1.536\nu^2\right)\left(\frac{t}{l}\right)^2}\right) \tag{3}$$

The material's shear modulus can be derived from the fundamental torsion frequency (f_t) as:

$$G = 4\,\frac{m\,f_t^2\,l}{w\,t}\left(\frac{\frac{w}{t} + \frac{t}{w}}{4\left(\frac{t}{w}\right) - 2.52\left(\frac{t}{w}\right)^2 + 0.21\left(\frac{t}{w}\right)^6}\right)\left(\frac{1}{1 + A}\right) \tag{4}$$

in which A is an empirical correction factor.

$$A = \frac{0.5062 - 0.8776\left(\frac{w}{t}\right) + 0.3504\left(\frac{w}{t}\right)^2 - 0.0078\left(\frac{w}{t}\right)^3}{12.03\left(\frac{w}{t}\right) + 9.892\left(\frac{w}{t}\right)^2} \tag{5}$$

2.2 Mixed numerical-experimental approach

Besides the use of standardized testing procedures, elastic material properties can also be identified with mixed numerical-experimental techniques (MNETs). Figure 1 displays the general flowchart of the MNET used to identify elastic material parameters. The experimental part consists of a modal analysis test performed on a freely suspended material specimen. This test configuration is used because it can be approximated by free-free boundary conditions in the finite element model. The measured resonance frequencies are used as input data for the identification routine. The numerical part of the method consists of a fully converged FE-model. The numerical frequencies are calculated using a set of trial values for the unknown material parameters. The numerical frequencies are compared with the measured frequencies, and corrected material properties are found by minimizing the residues of the frequency differences between the experimental and numerical frequencies. The improved material properties are inserted in the FE-model and a new iteration cycle is started. Once the numerical and experimental frequencies match, the procedure is aborted,

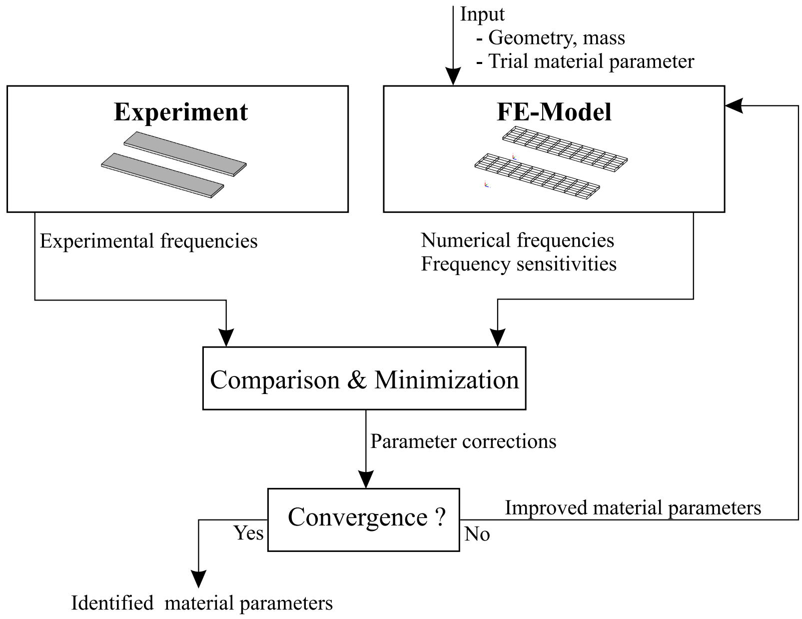

Figure 1: General flowchart for MNET based elastic material identification procedures.

and the desired material properties can be found in the database of the FE-model. Various applications have already shown that MNETs are a reliable tool to identify elastic material properties [3, 4].

Mathematically, the MNET identification procedure is formulated as an optimization problem. A sensitivity analysis provides the relation between an imposed parameter change and the resulting frequency shift:

$$[S]\{\Delta p\} = \{\Delta f\} \tag{6}$$

The vectors $\{\Delta p\}$ and $\{\Delta f\}$ contain the parameter and frequency changes, respectively. The matrix $[S]$ is the sensitivity matrix and contains the partial derivatives of the resonance frequencies with respect to the elastic material parameters. The elastic properties are found by minimizing the frequency differences between the experimental and numerical frequencies. The optimal frequency shift is thus defined by

$$\text{minimize} \;\; \|\{\Delta f\} - \{f_{num} - f_{exp}\}\|_2^2 \tag{7}$$

where $\| \diamond \|_2$ denotes the Euclidean norm. Inserting the sensitivity relation (6) into (7) provides the mathematical expression of the MNET's optimization problem.

$$\underset{\Delta p}{\text{minimize}} \;\; \|[S]\{\Delta p\} - \{f_{num} - f_{exp}\}\|_2^2 \tag{8}$$

To ensure a stable convergence of the iterative procedure, the optimization problem (8) is solved by considering a set of box constraints on the optimization parameters – the elements of the vector $\{\Delta p\}$ – in such a way that each material parameter cannot change more than 25$\%$ during one iteration step.

2.3 Layered materials

Due to the flexibiliy of finite element models, MNETs are an obvious choice to develop test procedures to measure the elastic properties of layered materials. The extension of non-layered to layered identification routines is discussed in detail in [5], where it is explained that it is impossible to identify the elastic properties of the layers of a laminate from the frequencies of one single specimen. Resonance frequencies of laminates are controlled by the overall stiffness of the material. But, since the same overall stiffness can be obtained with an infinite number of different layer stiffness combinations, it is impossible to decompose the overall stiffness into the correct layer stiffnesses. This uniqueness problem can be overcome by using the frequencies of a set of test specimens. Each specimen of this set must be made with the same material layers, but must have a different overall stiffness. In [5] it is proven that there will only be one set of layers stiffnesses that will result in the correct overall stiffness for all the test specimens, if the number of test samples is larger than or equal to the number of unknown material layers. In this way, the uniqueness of the solution can be assured.

The overall stiffness of the laminate can only be altered by changing the stacking sequence of the layers or by changing the thickness of – one of – the layers, which means that layered material identification requires a number of purpose built or modified samples. One type of layered materials where this approach can be followed are coated materials.

3 Uncertainty budgets

3.1 The analytical method for a homogeneous material

The UNCERT Code of Practice 13 [1] proposes a way to handle the uncertainty budget for the procedures described in the ASTM and EVN standards. This Code of Practice describes the relation between the uncertainty on the measurands (E, G) and measurements (m, w, l, t, f_f, f_t). For each of the measurements the uncertainty sources are identified (such as the uncertainty associated with the use of a caliper or balance) and classified according to Type A (evaluated by statistical means from a number of repeated observations) or Type B (obtained from calibration certificate, manufacturer's information or expert's estimation). In this paper, we will consider the identification of the properties of one sample with one system or tool for each of the measurements. The sample's shape is assumed to be well defined – as would be the case for carefully machined samples. Therefore, the measurements of the dimensions are highly repeatable, as is also known to be the case for the resonant frequencies. As a result the uncertainty contributions are all dominated by the instrument accuracy, which is an estimated uncertainty of Type B. The probability distributions are considered rectangular (true value certainly within range, with an equal probability in the whole uncertainty range). For this type of probability distribution, the standard uncertainty is obtained as the uncertainty range divided by the divisor $\sqrt{3}$. Assuming that the individual uncertainty sources are uncorrelated, the measurand's combined uncertainty, u_c, can be computed as

$$u_c = \sqrt{\sum_i \left(c_i\, u(x_i)\right)^2} \tag{9}$$

in which c_i is the sensitivity coefficient associated with the measurement quantity x_i. The sensitivity coefficients can be obtained by partial derivation of the identification formulas (1) and (4).

3.2 Mixed numerical-experimental approach

The application of (9) requires the sensitivity coefficients of the measurands with respect to the measured input quantities. Since MNET procedures are in nature numerical routines, it is impossible the obtain the sensitivity coefficients through analytical derivation. A direct numerical evaluation of the sensitivity coefficients, e.g. using a finite difference approach, would be very time consuming since a MNET procedure contains one or more finite element models in its iteration loop. A more practical approach is to linearize the MNET using the solution of the last iteration step as working point. Derivation of the sensitivities has then become straightforward since they are the coefficients of the linearized MNET equations.

3.2.1 Non-layered materials

Linearizing the frequency response surfaces of the finite element model by computing the first order Taylor approximation in the point defined by the obtained material parameters provides the relation of (10). Equation (10) describes the influence of a variation of the material parameters on the resonance frequencies of the test specimen,

$$\{\Delta f\} = [S_m]\{\Delta p\} \tag{10}$$

in which the vector $\{\Delta f\}$ contains the frequencies changes, the vector $\{\Delta p\}$ contains the applied parameter changes and $[S_m]$ is the sensitivity matrix. The relation of (10) can be inverted by using the pseudo-inverse of the sensitivity matrix. The inversion provides (11), which expresses the influence of a change of the frequencies on the identified material parameters.

$$\{\Delta p\} = [S_m]^{\dagger}\{\Delta f\} \tag{11}$$

Expression (11) thus allows to convert the uncertainties on the frequencies into uncertainties on the obtained material parameters. In the identification procedure the material parameters are obtained by comparing – and matching – the experimental and numerical frequencies. It is obvious that there is an uncertainty on the experimental frequencies, but there is also an uncertainty on the numerical frequencies. To construct the finite element model of the test specimen the specimen's geometry has to be measured. The uncertainties on the length, width, thickness and weight result in an uncertainty on the finite element model and thus in an uncertainty on the numerical frequencies. The frequency uncertainties consist of two parts: an experimental part $\{\Delta f_{exp}\}$ and a numerical part $\{\Delta f_{num}\}$.

$$\{\Delta f\} = \{\Delta f_{exp}\} + \{\Delta f_{num}\} \tag{12}$$

The uncertainties on the numerical frequencies can be related to the uncertainties on the geometrical parameters by means of the first order Taylor approximation of the frequency response surfaces in a working point defined by the value of the geometrical parameters of the finite element model. This process results in the relation of equation (13).

$$\{\Delta f_{num}\} = [S_g]\{\Delta G\} \tag{13}$$

Inserting the expressions of (12) and (13) into equation (11) provides a relation between the uncertainties on the inputs and outputs of the identification procedure.

$$\begin{aligned}\{\Delta p\} &= [S_m]^{\dagger}\Big(\{\Delta f_{exp}\} + [S_g]\{\Delta G\}\Big) && (14)\\ &= \underbrace{[S_m]^{\dagger}\{\Delta f_{exp}\}}_{\substack{frequency\\ contribution}} + \underbrace{[S_m]^{\dagger}[S_g]\{\Delta G\}}_{\substack{geometry\\ contribution}} && (15)\end{aligned}$$

By grouping the terms of (15) as

$$[\chi] = \Big[[S_m]^{\dagger}\ [S_m]^{\dagger}[S_g]\Big] \tag{16}$$

$$\{\Delta I\} = \begin{Bmatrix}\{\Delta f_{exp}\}\\ \{\Delta G\}\end{Bmatrix} \tag{17}$$

the influence of a change of the experimental frequencies and geometrical model parameters on a particular material property can be written as:

$$\Delta p_i = \sum_j \chi_{ij}\, \Delta I_j \tag{18}$$

Since χ_{ij} is the sensitivity coefficient of material property p_i with respect to the input parameter I_j, the combined uncertainty on p_i is given by

$$u_{p_i} = \sqrt{\sum_j \left(\chi_{ij}\, u(I_j)\right)^2} \tag{19}$$

3.2.2 Layered materials

The application of the presented uncertainty estimation method on the layered material identification routines requires two modifications: a) the MNET linearization will have to be extended to multi-model identification routines, b) the procedure has to be able to handle relations between the input uncertainties. The necessity of the first requirement is obvious. The second requirement is a result from the fact that the total sample thickness is the sum of the different layer thicknesses.

Multi-model routines The extension to multi-model routines is fairly straightforward. Consider a multi-model identification routine that uses resonance frequencies of n_s different test samples. For each of the n_s models, one can write

$$\left\{\Delta f^{\{i\}}\right\} = \left[S_m^{\{i\}}\right]\{\Delta p\}, \qquad \forall i \in 1,\ldots,n_s \tag{20}$$

in which the superscript $\diamond^{\{i\}}$ indicates that the i^{th} specimen is being considered. The equations for the different test samples can be combined as

$$\left\{\Delta f^{glob}\right\} = \left[S_m^{glob}\right]\{\Delta p\} \tag{21}$$

where $\{\Delta f^{glob}\}$ and $[S_m^{glob}]$ are the global frequency difference vector and sensitivity matrix for the material properties, respectively. $\{\Delta f^{glob}\}$ and $[S_m^{glob}]$ are both block vectors in which the i^{th} block line contains the data of the i^{th} specimen, or

$$\left\{\Delta f^{glob}\right\} = \begin{Bmatrix} \{f^{\{1\}}\} \\ \vdots \\ \{f^{\{n_s\}}\} \end{Bmatrix} \qquad \left[S_m^{glob}\right] = \begin{bmatrix} \left[S_m^{\{1\}}\right] \\ \vdots \\ \left[S_m^{\{n_s\}}\right] \end{bmatrix} \tag{22}$$

The influence of the geometrical errors on the frequencies of the finite element models of the different specimens are given by

$$\left\{\Delta f_{num}^{\{i\}}\right\} = \left[S_g^{\{i\}}\right]\left\{\Delta G^{\{i\}}\right\}, \qquad \forall i \in 1,\ldots,n_s \tag{23}$$

These n_s sets of equations can also be combined to one global set as

$$\{\Delta f_{num}^{glob}\} = [S_g^{glob}]\{\Delta G^{glob}\} \tag{24}$$

where the global vectors are given by

$$\{\Delta f_{num}^{glob}\} = \begin{Bmatrix} \left\{ f_{num}^{\{1\}} \right\} \\ \vdots \\ \left\{ f_{num}^{\{n_s\}} \right\} \end{Bmatrix}, \qquad \{\Delta G^{glob}\} = \begin{Bmatrix} \{G^{\{1\}}\} \\ \vdots \\ \{G^{\{n_s\}}\} \end{Bmatrix} \tag{25}$$

The global sensitivity matrix for the geometrical parameters is a block identity matrix with the following structure.

$$[S_m^{glob}] = \begin{bmatrix} \left[S_m^{\{1\}} \right] & \cdots & [0] \\ \vdots & \ddots & \vdots \\ [0] & \cdots & \left[S_m^{\{n_s\}} \right] \end{bmatrix} \tag{26}$$

In a similar way as in the single model case, it can be shown that

$$\Delta p_i = \sum_j \chi_{ij} \, \Delta I_j \tag{27}$$

where $[\chi]$ and $\{\Delta I\}$ have the same structure as specified by (16) and (17), but have to be calculated using the global vectors and matrices as defined by (22), (25) and (26).

Input uncertainty relations The uncertainty on the layer thicknesses of a layered material is a bit different than the uncertainty on the other input parameters. The uncertainties on the other parameters are completely independent, e.g. there is no relation between the uncertainty on the mass and the uncertainty on the length of the sample. However, the uncertainties on the layer thicknesses are not independent, since the sum of all the layer thicknesses has to be equal to the total thickness. Consider a beam with a thickness of 2 mm which has two layers with a thickness of 1 mm. Assume that the total thickness and the two layer thicknesses can all be measured with an uncertainty of 0.01 mm. This means that the layer thicknesses of both the layers can vary between 0.99 and 1.01 mm. But if the thickness of the first layer equals 0.99 mm, this thickness of the second layer can only vary between 1.0 and 1.01 mm. The thickness of the second layer cannot be smaller than 1.0 mm, since that would results in a overall thickness that is smaller than 0.99 mm. This clearly shows that the limits between which the thickness of one layer can vary depend on the thickness of the other layers.

Equation 9 is only valid for uncorrelated uncertainty sources, and is therefore no longer applicable. The analytical evaluation of correlated uncertainties is a highly complex matter. Since a computationally efficient linear approximation of the MNET procedure is available, it makes more sense to tackle the problem with Monte Carlo simulations. For each layer a thickness value is generated using a random number generator that produces a uniformly distributed set of test values that respects the upper and lower thickness bounds that are imposed on the considered layer. All the layer thicknesses are added to get total sample thickness. If the total sample thickness does not comply with the imposed bounds, the thickness value set has to be rejected. This process has to be repeated until a sample set with a sufficient number of correct thickness value sets has been obtained.

Calculating uncertainty budgets for layered materials The uncertainties on the experimental frequencies, length, width, and mass are uncorrelated. Their contribution to the measurand uncertainty can thus be computed with (9).

$$u_{uncorr.}(p_i) = \sqrt{\sum_j \left(\chi_{ij} \, u(I_j)\right)^2} \quad , \forall I_j \in \{f_{exp}, \, length, \, width, \, mass\} \tag{28}$$

The layer thickness uncertainties are correlated. The uncertainty contribution can be evaluated with a Monte Carlo simulation using n_{mc} test sample sets. Since the standard uncertainty is defined as one standard deviation, the uncertainty contribution of the layer thicknesses can be estimated as

$$u_{corr.}(p_i) = \sqrt{\frac{1}{n_{mc}-1}\sum_{q=1}^{n_{mc}}\Big(\sum_j (\chi_{ij}\;\Delta I_{j,q})\Big)^2} \quad ,\forall I_j \in \{layer\ thicknesses\} \tag{29}$$

in which q is the Monte Carlo test sample index. The total uncertainty is the combined uncertainty of both correlated and uncorrelated uncertainty sources, or

$$u(p_i) = \sqrt{(u_{uncorr.}(p_i))^2 + (u_{corr.}(p_i))^2} \tag{30}$$

4 Examples

Consider a set of three rectangular steel beams. The first beam is a pure uncoated sample. The two other beams are made of the same steel sheet, but are coated with a 0.2 mm and 0.4 mm coating, respectively. Table 1 gives an overview of the geometry of the three considered samples. Table 2 presents the properties of the steel and the coating material.

Table 1: The geometry of the considered test samples

			Thickness	
	Length (mm)	Width (mm)	Sub. (mm)	Coat. (mm)
Sample-1	100.0	20.0	1.500	–
Sample-2	100.0	20.0	1.500	0.200
Sample-3	100.0	20.0	1.500	0.400

Table 2: The properties considered materials

	E (GPa)	G (GPa)	ν (–)	density (kg/m^3)
Steel	200	80	0.30	7800
Coating	50	20	0.25	4000

The input quantities are determined using state-of-the-art frequency, length and mass measurement tools (respectively a digital vibration analysis system, a profile projector and an analytical balance). The associated uncertainties, which as explained in 3.1 are of Type B, are given in Table 3.

Table 3: The uncertainty on the input parameters

	l (mm)	w (mm)	t_{sub} (mm)	t_{coat} (mm)	t_{tot} (mm)	m (g)	f_f (%)	f_t (%)
Sample-1	0.02	0.01	0.005	–	0.005	0.001	0.1	0.1
Sample-2	0.02	0.01	0.005	0.010	0.005	0.001	0.1	0.1
Sample-3	0.02	0.01	0.005	0.010	0.005	0.001	0.1	0.1

4.1 Homogeneous steel beam

Table 4 summarizes the calculation of the uncertainty contributions for the pure steel sample in accordance to the UNCERT Code of Practice 13 [1].

Table 4: Uncertainties of the individual measurements and their relative contribution to the uncertainties of Young's and shear modulus.

	f_f (Hz)	f_t (Hz)	l (mm)	w (mm)	m (g)	t (mm)
value	797.47	2367.66	100.00	20.00	23.400	1.500
uncertainty*	0.80	2.37	0.02	0.01	0.001	0.005
divisor**	1.7321	1.7321	1.7321	1.7321	1.7321	1.7321
relative standard uncertainty (%)	0.0577	0.0577	0.0115	0.0289	0.0025	0.1925
sensitivity coefficient on E	2	–	3	1	1	3
sensitivity coefficient on G	–	2	1	1	1	3
rel. contrib. to uncertainty on E (%)	3.82	0.00	0.34	0.24	0.00	95.59
rel. contrib. to uncertainty on G (%)	0.00	3.84	0.04	0.24	0.00	95.88

* Estimated tool or method uncertainty, type B.
** For a rectangular probability distribution.

The data in Table 4 agree with the well-known observation that the uncertainty associated with the measurement of the thickness of the sample is crucial. Table 5 provides the MNET sensitivity coefficients for the elastic and shear modulus.

Table 5: Sensitivity coefficients for the MNET routine.

	f_f (Hz)	f_t (Hz)	l (mm)	w (mm)	m (g)	t (mm)
sensitivity coefficient on E	2.0515	0.0096	3.0740	1.0368	1.0034	3.0586
sensitivity coefficient on G	0.1177	2.1311	1.0037	0.9397	0.9895	2.9502

With the input data of Table 4 and 5, the combined relative standard uncertainty and the expanded uncertainty (for a coverage factor $k = 2$ corresponding to a confidence level of 95%) can be computed with both the standard (9) and MNET (27) procedure. Both methods yield the same results. The results shown in Table 6 confirm that the vibration based methods have a very high degree of accuracy.

Table 6: Combined and expanded relative uncertainties of Young's and Shear modulus of a homogeneous material.

	E	G
combined relative uncertainty (%)	0.59	0.59
expanded relative uncertainty ($k = 2$, confidence level of 95%) (%)	1.18	1.18

4.2 Coated steel beams

The elastic and shear modulus of the steel substrate and coating can be derived from the fundamental flexural and torsion frequency of the considered samples. Since there are two layers, the layered identification algorithm requires the frequencies of at least two samples. Using the provided samples, the material properties can be obtained with the four different sample combinations of table 7.

Table 7: The four possible sample sets

	Sample-1	Sample-2	Sample-3
Set-1	$\star$	$\star$	–
Set-2	$\star$	–	$\star$
Set-3	–	$\star$	$\star$
Set-4	$\star$	$\star$	$\star$

These four sample combinations all result in the same values for the material parameters, but they do not identify the properties with the same uncertainty. For the coated samples the measurand uncertainties can be calculated with (30). By using the input uncertainties of Table 3, the uncertainty intervals of figure 2 are obtained.

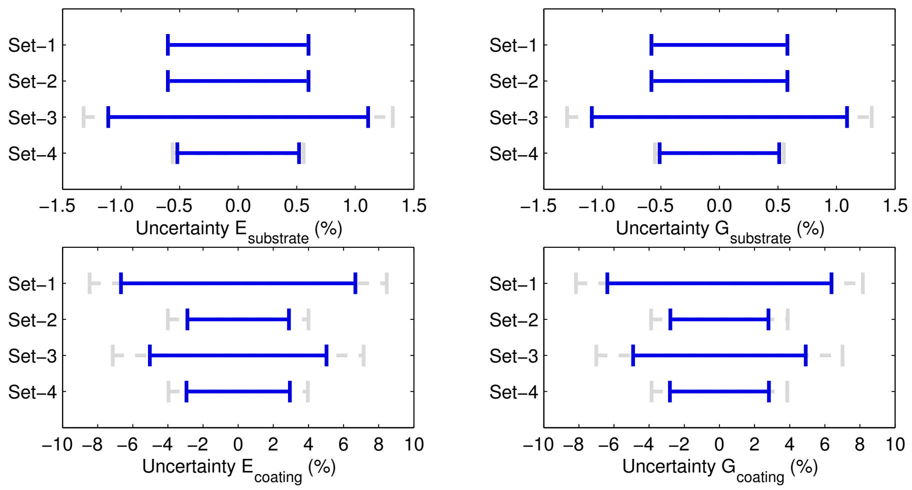

Figure 2: The standard combined uncertainty for the elastic properties identified on the coated steel beams.

The solid lines of the plots of figure 2 represent the correct results, i.e. the results obtained by taking the relations between the sample and layer thicknesses into account. Comparison of the results of the first two sample sets shows that a thicker coating results in a lower uncertainty on the coating properties, but does not affect the uncertainty on the substrate properties. Also note that the uncertainties on the substrate properties are the same as the uncertainties found in the previous example. This indicates that the uncertainty on the substrate properties is entirely controlled by the uncertainty on the pure substrate sample. The uncertainties on the coating properties are influenced by both samples.

The third sample set is the only set that does not include a pure substrate sample. It results in the highest substrate uncertainty and second highest coating uncertainty. It is thus advisable to include a pure substrate specimen in the sample set. The use of a pure substrate appears to increase the reliability of the results, as is clearly illustrated by comparing the uncertainties of samples sets 3 and 4. Extending the third sample set with a pure substrate – sample set 4 – results in a substantial reduction of the uncertainty of both the substrate and coating properties.

The dashed lines in figure 2 represent the uncertainties that were obtained when ignoring the relations between the uncertainties on the sample and layer thicknesses. The results clearly show that these relations have to be taken into account in order to obtain meaningful results, e.g. for set-4 this results in an overestimation of the uncertainty intervals with one third.

5 Conclusions

A method to estimate the uncertainty on the material parameters identified with mixed numerical experimental techniques was presented. The method estimates the uncertainty of the material parameters from the uncertainty on the input parameters. The proposed method has two important uses. Besides quantification of the uncertainty on the parameters obtained with a particular test, it can also be used in a pretest phase to compare a number of possible test setups. For example, the identification of a layered material requires a set of test specimen with different layer thicknesses. To find the most optimal sample set, the proposed method can be used to estimate the uncertainties on the obtained parameters for all the considered sample sets. The set which results in the lowest uncertainty on the identified parameters is obviously the preferred sample set.

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