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Fast Solutions in FSI-Problems using CMS-Methods

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Parameter studies in FSI-problems may often become quite time consuming. In most cases the fluid parameters are well defined and only the influence of the parameters of the solid component is subject to investigate. Therefore it would be desirable to investigate the fluid and solid part separately and finally combine them using CMS-Methods. This approach would also provide a good physical insight into the individual and combined behaviour of the components fluid and solid. Generally for each interface DOF one constraint mode must be added to the modal base. Since in most problems in FSI the fluid-structure interface involves many DOF the general CMS approach becomes inefficient. To reduce the number of constraint modes it is proposed to use the mode shapes of each component as a load function on the other domain. The static solution provides the modal based attachment modes (MAM). Their number corresponds to the number of total component modes which is in most cases much less then in the classic approach. The application is shown in optimization, updating, and a parameter study.

1 Introduction

Considering structure and fluid as components in an FSI-problem in a FE-analysis, the idea to apply CMS is obvious. In order to describe the interaction forces each CMS methods requires a number of constraint modes. Usually their amount depends on the amount of interface DOF. Since those are quite numerous in typical FSI problems a more efficient description of the interaction by a small amount of form functions would be desirable. An efficient method is proposed where the constraint modes are formed on the base of the modal DOF of each decoupled subsystem (Modal based Attachement Modes, MAM).

2 Application of CMS in FSI

Fluid-Structure-Interaction (FSI) can be described by a coupled system of equations (1) in a Finite-Element formulation with the mass matrix M and a stiffness matrix K for the components solid (displacement w as degree of freedom) and the component fluid (pressure p as degree of freedom). The matrices K_{SF} und M_{FS} establish the coupling of the individual components.

$$\begin{bmatrix} \mathbf{K}_{SS} & \mathbf{K}_{SF} \\ \mathbf{0} & \mathbf{K}_{FF} \end{bmatrix} \begin{Bmatrix} \mathbf{w} \\ \mathbf{p} \end{Bmatrix} + \begin{bmatrix} \mathbf{M}_{SS} & \mathbf{0} \\ \mathbf{M}_{FS} & \mathbf{M}_{FF} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{w}} \\ \ddot{\mathbf{p}} \end{Bmatrix} = \begin{Bmatrix} F \\ \rho A \ddot{w}_n \end{Bmatrix} \tag{1}$$

Modal transformation offers advantages if the modal base can be reduced to a set of few mode shapes used for transformation to a system with few modal DOF (2).

$$\begin{bmatrix} \mathbf{\Phi}_S^T \mathbf{K}_{SS} \mathbf{\Phi}_S & \mathbf{\Phi}_S^T \mathbf{K}_{SF} \mathbf{\Phi}_F \\ \mathbf{0} & \mathbf{\Phi}_F^T \mathbf{K}_{FF} \mathbf{\Phi}_F \end{bmatrix} \begin{Bmatrix} \mathbf{w}^* \\ \mathbf{p}^* \end{Bmatrix} + \begin{bmatrix} \mathbf{\Phi}_S^T \mathbf{M}_{SS} \mathbf{\Phi}_S & \mathbf{0} \\ \mathbf{\Phi}_F^T \mathbf{M}_{FS} \mathbf{\Phi}_S & \mathbf{\Phi}_F^T \mathbf{M}_{FF} \mathbf{\Phi}_F \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{w}}^* \\ \ddot{\mathbf{p}}^* \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Phi}_S^T \mathbf{F} \\ \rho A \mathbf{\Phi}_F^T \ddot{\mathbf{w}}_n \end{Bmatrix} \tag{2}$$

To compensate the truncation error it is common practice to integrate additional attachment modes $\mathbf{\Psi}$ into the modal base $\mathbf{\Phi}$ (3), which include the quasi static contribution of the dynamically not active mode shapes of the higher eigenfrequencies.

$$\mathbf{w} = \sum_{i=1}^{n_{red}} \mathbf{\Phi}_{S\,i} w_i^* + \sum_{j=1}^{k} \mathbf{\Psi}_{S\,att\,j} w_j^*$$
$$\mathbf{p} = \sum_{i=1}^{n_{red}} \mathbf{\Phi}_{F\,i} p_i^* + \sum_{j=1}^{k} \mathbf{\Psi}_{F\,att\,j} p_j^* \tag{3}$$

Since the topology of interaction is not predictable two form functions must be provided for each DOF at the interface in classic CMS-Methods („free“, „constrained“, Craig-Bampton etc.). By their superposition with the modal bases of the components the interaction topology can be closely approximated. The classic methods are therefore advantageous as long as the number of interaction DOF is small, since then also the amount of additional constraint modes remains small (application in [3]).

3 Generation of MAM

3.1 Procedure

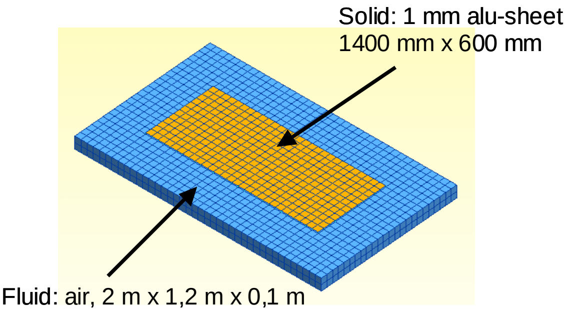
The number of interaction DOF between a fluid and a solid is often considerably high. Specifically when modelling a simple plate resonator in a room to simulate room acoustics the amount of interaction DOF corresponds to the number of nodes of the shell elements of the resonator. Classic CMS becomes inefficient in this case.

In a first step it is assumed, that the interacting forces in the interface can be approximated by a superposition of the unconstrained modes of the corresponding attached component. In a next step the modes of one component are considered as Neumann boundary conditions for the other component at the interface. Analogically to the mode acceleration method or the modal augmentation method attachement modes are computed by solving a static equation (4). Instead of DOF-related “attachment modes” form functions are computed which refer to modal DOF (MAM). In the special case of FSI this approach is applicable since an interaction between Neumann and Dirichlet boundary condition is established in the system of equations in the interface.

$$\begin{aligned} \begin{bmatrix} \mathbf{K}_{solid} \end{bmatrix} \begin{Bmatrix} \Psi_{attS,1} & \cdots & \Psi_{attS,n} \end{Bmatrix} &= \begin{Bmatrix} \mathbf{0} \\ \mathbf{\Phi}_{fluid}\big|_I \end{Bmatrix} \\ \begin{bmatrix} \mathbf{K}_{fluid} \end{bmatrix} \begin{Bmatrix} \Psi_{attF,1} & \cdots & \Psi_{attF,n} \end{Bmatrix} &= \begin{Bmatrix} \mathbf{0} \\ \mathbf{\Phi}_{solid}\big|_I \end{Bmatrix} \end{aligned} \tag{4}$$

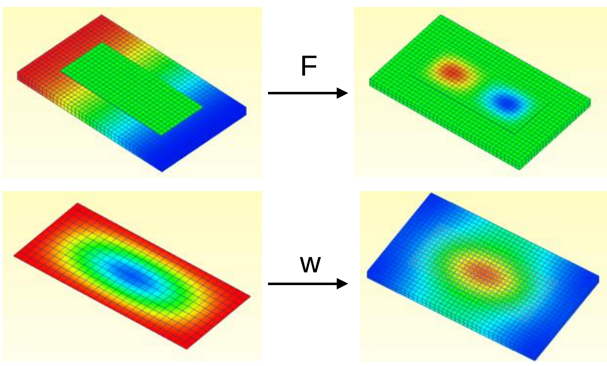
3.2 Validation example: plate resonator

Fig. 1 shows a closed volume of air with a plate structure with hinged boundary conditions. In total ten decoupled modes were computed in a frequency range between 70 … 200 Hz. To compute the MAM’s each mode of one system was used as a Newman boundary condition of the other system at the interface. The modes of the fluids are used as a pressure load on the plate and the modes of the plate as an acceleration load on the fluid.



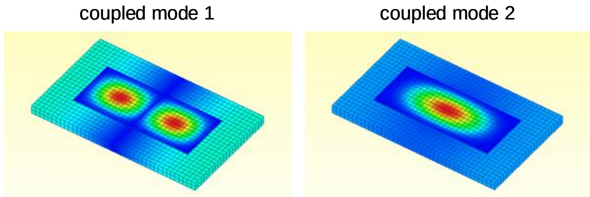
.Fig.1 FE-model of a plate resonator coupled to a fluid volume

The static solution vectors of the plate displacement is orthogonalized to the existing normal modes and to each other and then added to the modal base of the plate. Accordingly the static fluid pressure vectors obtained by the static solution of the fluid component are also orthogonalized and added. To get the static solution of the unconstrained fluid first needs a compensation of acceleration “load” vector by the rigid body mode. Otherwise the solution would be singular.



.Fig.2 Generation of MAM, top: static deflection shape of the plate resulting from the pressure field of mode 1 of the fluid; bottom: “static” pressure by applying an acceleration vector of the 1st mode of the plate with hinged bc.

Then the total modal base contains 10 normal modes + 10 MAM. In table 1 the first 10 natural frequencies are compared to the approximated solution in modal domain.



.Fig.3 FSI, coupled modes

While the solution with MAM corresponds almost exactly to the reference solution of the total coupled system of equations, the solution without MAM differs by up to 4.3 %. The same result could of course be obtained by applying classic CMS. In this case 377 attachement modes would have to be computed. Even with 20 modes the error does not become significantly smaller without additional constraint modes. It would take 100 modes and more to approximate the reference solution sufficiently.

Mode	Reference	10 Modes + 10 MAM's			10 modes w/o MAM's		
Nr.	Hz	Hz	Diff. %	MAC	Hz	Diff. %	MAC
1	69.7	69.7	0.0	100	71.3	2.2	99.2
2	79.2	79.2	0.1	100	82.4	4.1	99.1
3	98.7	98.7	0.0	100	102.7	4.0	98.5
4	112.5	112.5	0.0	100	117.4	**4.3**	96.5
5	137.0	137.0	0.0	100	137.3	0.3	99.9
6	152.1	152.1	0.0	99.9	157.0	3.2	94
7	161.0	161.2	0.1	100	162.4	0.9	99.8
8	173.6	173.6	0.0	100	175.1	0.9	99.5
9	188.8	190.5	0.1	99.9	208.7	2.7	85.5
10	203.3	203.4	0.1	99.1			
Average			**0.0**	**99.9**		**2.5**	**96.9**

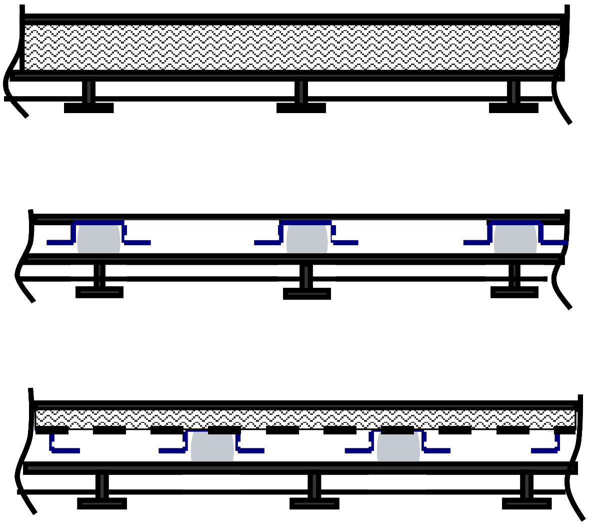
Mode	Reference	w/o MAM's, 20 modes			w/o MAM's, 100 modes		
Nr.	Hz	Hz	Diff.	MAC	Hz	Diff.	MAC
1	69.7	70.4	1.0	99.8	69.9	0.3	100
2	79.2	81.0	2.4	99.6	79.6	0.6	100
3	98.7	100.4	1.7	99.7	99.1	0.4	100
4	112.5	116.6	**3.7**	97.4	113.2	**0.6**	100
5	137.0	137.2	0.1	100	137.0	0.0	100
6	152.1	156.3	2.8	96.5	153.0	0.6	99.9
7	161.0	161.1	0.1	100	161.0	0.0	100
8	173.6	174.6	0.6	99.8	173.7	0.1	100
9	188.8	193.4	2.5	95	189.8	0.6	99.8
10	203.3	208.2	2.4	91.8	204.6	0.7	99.6
Average			**1.7**	**98.0**		**0.4**	**99.9**

Table 1 Eigenfrequencies, error in % and MAC, correlation of the solution by CMS with and without MAM to reference.

4 Application

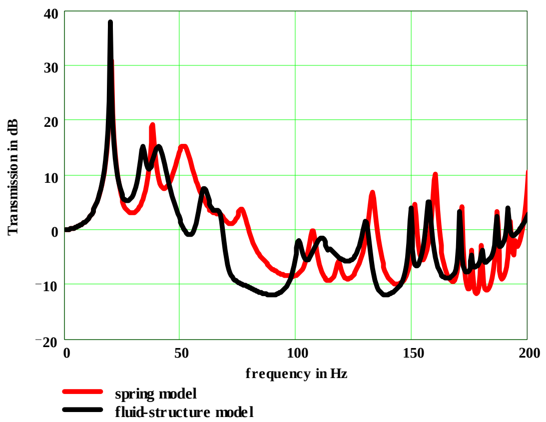
4.1 Parameter study of ship floors

Fig.4 shows the principle design of three different types of floating floors in ship floor structures. Type 1 includes a classic layer of mineral wool. In type 2 and 3 the top plates are mounted on descrete elastomer elements to achieve lower tuning frequencies of the floor system. Type 3 combines the springs with an additional layer of mineral wool.



.Fig.4 FSI, coupled modes

The transmission of structure borne sound was studied by simulation. Within this study the effect of the closed air volume between the plates was not clear. Typical models simply add the vertical stiffness of the enclosed air cushion to the stiffness of the isolating material. This approach however neglects the effect of transversal modes of the fluid volume. In a parameter study of similar systems this effect may be significant. Therefore a second model was created modelling explicitly the air volume and assuming FSI. To avoid excessive computational time in a repetitive parameter study, an evaluation in modal domain was extremely efficient using MAM.

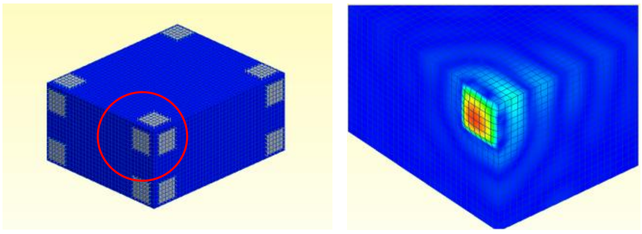


.Fig.5 Spectra of transmission of structure borne sound in dB, comparison of an FSI-model and a substitute model with spring elements

Fig. 5 compares the transmission spectra of both approaches, with and w/o FSI. Within the critical frequency range between 40 … 100 Hz the differences are significant.

4.2 Optimization of a reverberation room

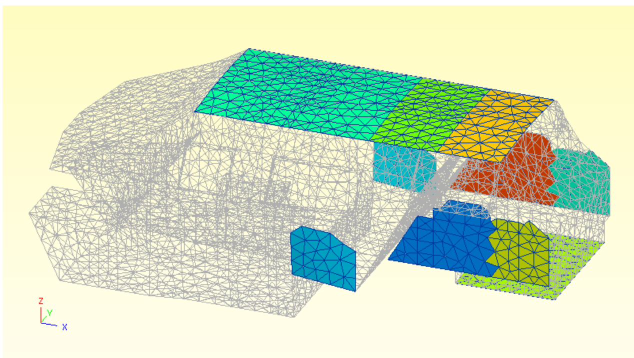
Figure 6 shows the FE-model of a reverberation room equipped with 12 possible positions of plate resonators. Four resonators shall be installed to simulate a larger room size in the lower frequency range. In a first step the volume is modelled. The modes of the fluid and for all possible resonators are computed separately. This is equally done for the MAMs. The 495 possible combinations are then computed in modal domain which is simply set up by activating the modes of the respective resonators and their constraint modes. The FRF are then examined by modal superposition.



.Fig.6 FE-model of a reverberation chamber with 12 possible resonator positions (left) and one orthogonalized “static” MAM of the fluid component.

4.3 Updating of a car interior

Figure 7 shows a set of plate resonators of a car interior. Their natural frequency is tuned by the stiffness of the spring elements supporting the resonators. The residue between measured and computed frequency response functions is minimized by updating of the spring stiffnesses. Also in this case the modal base of the car interior must only be computed once. Since the FRF have to be recalculated for each updating iteration, an analysis in a reduced modal base was extremely time saving.



.Fig.7 FE-Model of a car interior with plate resonators

5 Conclusion

CMS methods are specifically suited for optimization and updating tasks as long as the number of DOF at the component interface is small in relation to the total amount of DOF. However in FSI the number of interface DOF is generally not small when dividing the structure into a solid and a fluid component.

Involving constraint modes based on modal DOF (MAM) is highly efficient. Modal analysis is performed on symmetric matrices. In a reduced modal base the computational time is decreased to a fraction of the time which is consumed by the solution of the total system of equations with unsymmetric matrices.

Acknowledgments

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