

# A STUDY ON DAMAGE DETECTION USING OUTPUT-ONLY MODAL DATA

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## ABSTRACT

This paper will describe the results of vibration studies conducted during the second phase of the activities of the ASCE Structural Health Monitoring Task Group<sup>1</sup>. These activities focus on the application of damage detection techniques to experimental data. Also, this paper will focus on damage detection using output-only data from the vibration study. A one-third scale model of a four story steel frame at the University of British Columbia was used as the test specimen. A series of forced and ambient vibration tests on this frame for various levels of damage were conducted. Damage was simulated by removing members within the structure. The natural frequencies and their associated mode shapes were determined for each damage case using frequency-domain and time-domain techniques. A finite element model of the structure was updated using output-only modal identification results from the vibration measurements of each damage case. Finally, the damage was determined from the changes in the element properties resulted from the model updating process of the finite element model benchmark.

## INTRODUCTION

During the past few decades, a significant amount of research has been conducted in the area of non-destructive damage evaluation (NDE) based on changes in dynamic properties of a structure. Each of the NDE methods developed to date can be classified into different levels according to their performance and application. This paper will focus on the application of one of the levels of the NDE which is the non-destructive damage detection techniques being applied to experimental data. This study is part of the second phase of the activities of the ASCE Structural Health Monitoring Task Group. To provide the experimental data, a one-third-scale model of a four story steel frame at the University of British Columbia was used as the test specimen. A series of forced and ambient vibration tests on this frame for various levels of damage were conducted on July 19-21, 2000. Progressive damage was simulated by removing bracing from the structure and loosening the connections. For the forced vibration tests an electromagnetic shaker was used to excite the

structure at the top floor. Accelerometers placed throughout the structure were used to measure the structural response. For the ambient vibration tests the shaker was turned off and the ambient vibration of the structure was recorded for several minutes. The results of a study on non-destructive damage detection using data from the ambient vibration tests to perform automated correlation analyses between experimental and analytical models are presented in this paper. The FE model of the "undamaged" structure was updated with results from ambient vibration tests of the undamaged model. Then the results of the modal identification of each of the "damaged" cases were used to perform correlation analyses with the updated FE model of the undamaged structure. The sensitivity of selected parameters used for the correlation study was assessed and those parameters that showed highest sensitivity were associated to changes on the structure due to the induced damage. Of the five different damage cases investigated, four cases were successfully predicted. For the case where damage was not properly identified it was found that the induced damage was not significant enough to produce noticeable changes in the modal properties of the structure, and thus the sensitivity analyses were not able to provide a reliable identification of the presence of damage.

## DESCRIPTION OF FRAME MODEL

A modular four storey, two by two bay, steel frame structure has been designed and built by the Earthquake Engineering Research Laboratory at the University of British Columbia (Fig. 1). The model is approximately 3.6 m tall with a total width of 2.5 m. Each floor is 0.9 m high and each bay is 1.25 m wide. For more information on the frame see Black and Ventura (1998), (Ref. [1]).

The applied load on the first, second and third floor of the steel frame was chosen to be each approximately 17.8 kN and for the roof level (fourth floor) about 13.4 kN. To simulate this uniformly distributed load, several massive steel plate elements were placed on each bay per floor. Dimensions of the steel plates are 1.5 x 0.65 x 0.06 m for the first three floors and 1.5 x 0.65 x 0.045 m for the roof level. The weight of the electromagnetic shaker installed at the roof level is about 2 kN.

<sup>1</sup> For further information please visit  
(<http://wusceel.cive.wustl.edu/asce.shm/EMD2000.htm>)

## VIBRATION TESTING

To determine the vibration characteristics of the steel frame such as the natural frequencies and mode shapes, ambient and forced vibration testing methods were applied. Unlike forced vibration testing, the force applied to a structure in ambient vibration testing is not controlled. The measurements, in our case accelerations, are taken for a long duration to ensure that all the modes of interest are sufficiently excited.

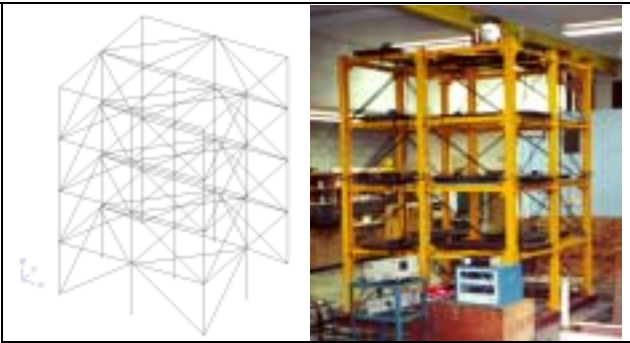


Figure 1. Steel Frame Model, a) FEM wire frame model (left); b) actual unit as tested (right)

The forced vibration testing simply consisted of a low amplitude vibration introduced by an electromagnetic shaker installed at the roof level. The shaker is a Ling Dynamic Systems 450 Series Vibrator, which connected to a Ling Power Amplifier (PA-1000) (For further information on the shaker see Kharrazi 2001 Ref.[2]). The shake level chosen for the test was full and half the maximum magnitude of the shaker force. A digital wave generator (3525 Dual Channel FFT Analyser) produced random vibration (white noise) with a frequency band of 0.1 to 50 Hz. The spectra for the generated white noise signal ascended with a relatively slow ramp, which was from about 0.1 Hz to 1 Hz.

The electromagnetic shaker was installed at the roof, on top of a steel plate, at a 45 degree angle off the main direction of the frame. To capture the induced force to the frame, the acceleration and displacement of the shaker was recorded. Based on the recorded data, the vibrator generated a maximum acceleration of about 5.0 g and a maximum force of about 200 N for the full-amplitude setups. The same weight created a maximum acceleration of 2.25 g and a maximum force was approximately 90 N for the half amplitude setups.

Three different vibration measurement systems were utilized for this project. These are described thoroughly in Ref. [2]. For the tests conducted on the steel frame, each dataset was collected for 6 minutes. One of the systems collected the data at a sampling rate of 2000 samples per second and decimated to 250 sps, for storage purposes. Fourteen accelerometers were used for the ambient vibration measurements. Only one setup was necessary to capture the natural frequencies and mode shapes. Measurements were taken in three locations on every floor

beginning from the roof down to the 1st floor. Finally, measurements for all three directions were taken in one location at the base level, which was the shake table surface. Figure 2 shows a typical accelerometer layout of the approximate locations and the directions of accelerometers on each floor level.

DasyLab<sup>®</sup> Version 5.01.10 (DasyLab<sup>®</sup> User's Guide, Iotech, 1998) was one of the programs used to record the forced vibrations and ambient vibration of the frame.

The computer program ARTeMIS<sup>®</sup> Extractor Version 2.0 (Structural Vibration Solution ApS (<http://www.svibs.com>)) was used to identify the natural frequencies and mode shapes of the structure. The data was analysed using both the Frequency Domain Decomposition (FDD) (frequency domain analyses) and the Stochastic Subspace Iteration (SSI) (time domain analyses) options included in ARTeMIS<sup>®</sup>.

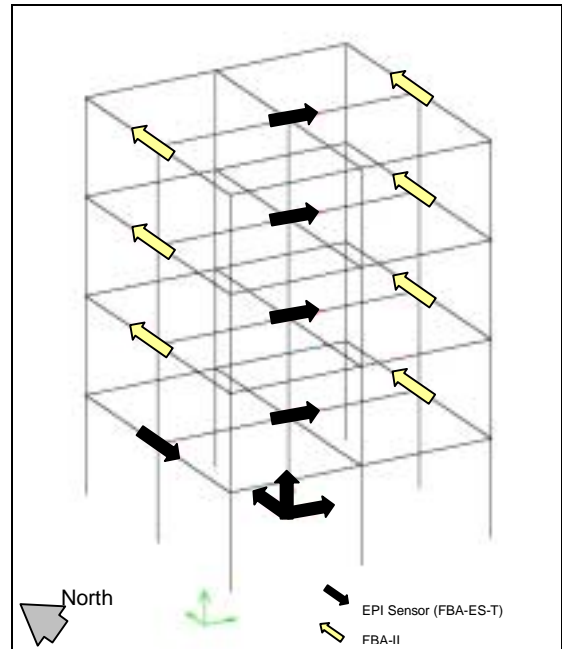


Figure 2. Typical accelerometer locations and directions on the steel frame

## TEST DESCRIPTION

The vibration measurements were performed on July 19-21, 2000 with ten test configuration cases. All test configurations are conducted at the full and half amplitude forced vibration levels. In addition to the forced vibration testing, ambient vibration testing was performed for each case. The damage in each case was introduced by removing brace elements or disconnecting the beam-column bolted connections. The configuration cases are as follow:

**Case I**, The configuration of the steel frame structure for the first case was an undamaged structure. The

undamaged steel frame was measured for ambient vibration and with full and half amplitude for forced vibration.

**Case II**, To introduce mass asymmetry to the steel frame structure, four steel plates were added to the first floor and two to the second floor. Each steel plate added to the first floor weighed 0.25 kN with an average dimensions of 35 x 57 mm. Each of the steel plates added to the second floor weighed 0.25 kN with similar dimensions as the above-mentioned plates. The added amount of mass is 2.8% of the 2nd floor's mass and 5.6% of the 1st floor's mass. These loads were added in the far south side of the frame, aligned to the large steel plate elements in that floor.

**Case III**, Damage was introduced to the frame by removing one brace from the north west corner of the frame. The eliminated brace was attached to the steel base of the column and the first floor. The removed brace is shown in Fig. 3, indicated with the number 1.

**Case IV**, In the next damage case a second brace was removed. This brace was eliminated from the northwest side bay between the 2nd and 3rd floors. The removed brace is shown in Fig. 3, number 2.

**Case V**, The third damage condition was introduced to the frame by disconnecting the beam-column connection in the north west corner of the first floor in addition to the previous removals. The disconnected connection is indicated with number 3 in Fig. 3.

**Case VI**, The fourth case featured all of the braces removed and the disconnected joint in the previous case were re-attached. The structure was measured with no damage and without any brace elements.

**Case VII**, Damage was induced on the same beam disconnected in case V. The beam to column connection of the 1st floor in the northwest corner was disconnected. This location is displayed as point 3 in Figure 3.

**Case VIII**, In addition to the damage in case VII, the beam to column connection in the north side of the 1st floor was partially loosened. This was done on the next bay to the damage created in case VII. The loosened joint is shown in Fig. 3 as number 4.

**Case IX**, All of the connections were retightened to "repair" them to the original state. The frame had the same configuration as in case VI.

**Case X and Case IX**, were redone to control the re-testing ability in ambient and forced vibration testing.

## NATURAL FREQUENCIES AND MODE SHAPES

The lateral and torsional natural frequencies and mode shapes were estimated using the 14 ambient vibration measurements collected. The torsional frequencies were estimated from the difference in the measured lateral motions obtained from opposite sides of the frame. This

was possible as the deck was assumed to be rigid and, therefore, the difference in the motion at each side of building give a reasonable estimate of the torsion. The steel plate elements and the diamond bracing of each floor justified this assumption.

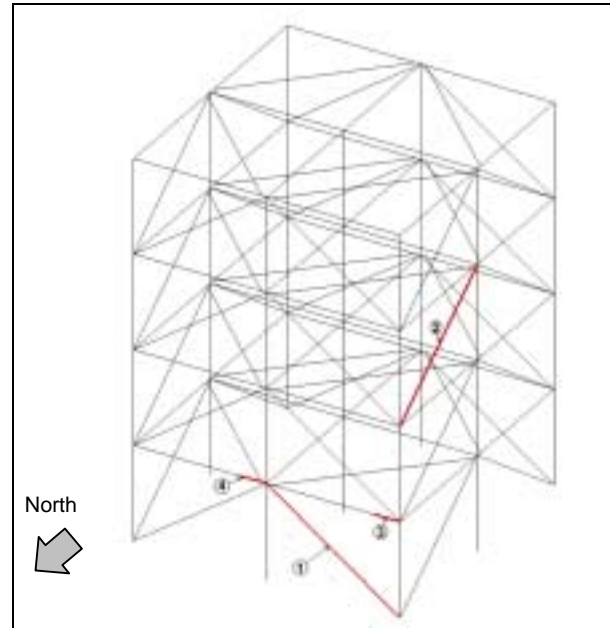


Fig. 3. Damage introduced to the steel frame in different cases

The mode shapes were generally well defined in the east west direction. In the north-south direction, the higher modes shapes were difficult to define clearly.

The magnitudes of averaged normalized singular values of spectral densities from frequency domain decomposition of the above measurements were calculated using ARTeMIS<sup>®</sup> to identify the natural frequencies of the structure. The data was decimated by the factor of 2 and the frequency resolution is set to 1024 frequency lines.

The ambient vibration records obtained were used to determine the transfer function, coherence and phase between the reference sensors and all other sensors. This information was used to confirm if each peak in the FDD corresponded to a natural frequency or to an operating mode of vibration at that frequency. Table 1 show the frequencies (and periods) that were determined to be the natural frequencies of the frame in different cases.

One of the mode shapes determined to be natural modes of vibration of the frame is shown in figure 4. The mode shape is shown in three views: an isometric view at the top left of the figure, an elevation view at the top right and bottom left and a plan view at the bottom right.

The forced vibration testing as mentioned earlier is conducted with the use of an electromagnetic vibrator. Since the motion shape was a random vibration

(essentially white noise) and it was assumed as an unknown variable, the same analyses that have been performed previously for the ambient vibration testing were done for the forced vibration testing. Because of a limitation in the produced vibration, natural frequency content of the steel frame close to 1 Hz are not well defined.

The mode shapes determined to be natural modes of vibration of the frame are the same as obtained by the ambient vibration testing. The results of the analyses are summarized in Table 1.

### DAMAGE DETECTION AND CORRELATION ANALYSES

During the past decades, a significant amount of research has been conducted in the area of non-destructive damage evaluation (NDE) based on changes in dynamic properties of a structure. Each of the NDE methods developed to-date can be classified into one of four levels according to their performance (Ref. [3]).

- 1. **Level I** - those methods that only identify if damage has occurred.
- 2. **Level II** - those methods that identify if damage has occurred and simultaneously determine the location of the damage.

3. **Level III** - those methods that identify if damage has occurred and simultaneously determine the location of the damage and as well as estimate the severity of the damage.

4. **Level IV** - those methods that identify if damage has occurred and simultaneously determine the location of the damage, estimate the severity of the damage, and evaluate the impact of damage on the structure.

A level two non-destructive damage evaluation technique was performed in this study using the results from ambient vibration testing.

To detect the damage created to the frame, a finite element program was used to model the structure. The software, FEMtools<sup>®</sup> Version 2.0 (Dynamic Design Solutions N.V. (DDS) <http://www.dds.be>) was used for this purpose. One of the main objectives of integrating test and analysis in FEMtools<sup>®</sup>, is to compare numerically and experimentally obtained data. Correlation analysis is one of the collection of methods that are available in FEMtools<sup>®</sup> to compare two sets of data, usually one from the analytical database and another from the experimental one. Analysis options such as spatial correlation, shape correlation, shape pairing, FRF pairing, FRF correlation functions and correlation coefficients calculation are available in FEMtools<sup>®</sup>.

Table 1. Modes Determined Below 40 Hz, ambient and forced vibration with half and full amplitude level, for all cases

Mode	Freq. from Ambient Vibration Testing (Hz)					Frequency from Force Vibration Testing										Description
						Full Amplitude Level (Hz)					Half Amplitude Level (Hz)					
	Case	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	
1	4.69	4.69	4.59	4.49	4.40	4.64	4.64	4.59	4.44	4.40	4.69	4.69	4.59	4.49	4.44	First N/S Mode (1NS)
2	4.98	4.98	4.88	4.88	4.88	4.93	4.93	4.83	4.83	4.88	4.93	4.93	4.83	4.83	4.88	First E/W Mode (1EW)
3	10.35	10.35	10.16	9.77	9.47	10.35	9.86	9.81	9.62	9.57	10.30	10.30	9.86	9.62	9.62	First Torsional (1T)
4	12.70	12.70	12.50	12.21	11.91	12.11	12.11	12.11	11.67	11.67	12.55	12.35	12.55	11.91	11.91	Second N/S Mode (2NS)
5	15.04	15.04	15.04	15.04	15.04	14.84	- <sup>2</sup>	-	13.72	13.72	14.99	14.99	14.84	14.40	13.96	Second E/W Mode (2EW)
6	23.73	23.73	23.73	23.73	23.73	-	23.34	22.85	22.66	22.66	23.54	23.34	23.34	23.10	22.85	Third E/W Mode (3EW)
7	24.32	-	24.32	24.22	24.22	-	24.22	24.02	24.02	24.02	-	24.27	24.22	24.02	23.97	Fourth E/W Mode (4EW)
8	34.18	34.18	34.18	33.89	33.79	34.08	32.08	31.79	31.79	31.79	34.52	34.52	34.52	34.52	34.28	Fifth E/W Mode (5EW)
9	39.94	39.94	39.94	39.94	39.94	-	-	39.79	39.55	39.31	40.23	40.23	40.23	39.79	39.31	Third N/S Mode coupled with Torsion (2EW + T)
Case	VI	VII	VIII	IX	X	VI	VII	VIII	IX	X	VI	VII	VIII	IX	X	
1	1.66	1.66	1.66	1.66	1.66	-	-	-	-	-	-	-	-	-	-	First N/S Mode (1NS)
2	2.83	2.83	2.83	2.83	2.83	-	-	-	-	-	2.83	2.83	2.59	2.83	2.88	First East West Mode (1EW)
3	3.32	3.32	3.32	-	3.32	2.83	2.83	2.69	2.83	2.88	-	-	-	-	-	First Torsional (1T)
4	5.57	5.57	5.57	5.57	5.57	-	-	-	-	-	5.57	5.57	5.57	5.62	5.66	Second N/S Mode (2NS)
5	9.08	9.08	9.08	9.08	9.08	5.57	5.57	5.57	5.57	5.62	9.28	9.28	9.13	9.33	9.38	Second E/W Mode (2EW)
6	10.35	10.35	10.35	10.35	10.35	9.18	9.18	8.94	9.23	9.33	10.30	10.30	10.25	10.30	10.45	Second Torsional (2T)
7	10.84	10.84	10.84	10.84	10.84	10.25	10.25	10.06	10.25	10.25	10.79	10.74	10.74	10.79	10.79	Third N/S Mode (3NS)
8	15.04	15.04	15.04	15.04	15.04	10.74	10.74	10.55	10.74	10.74	15.77	15.77	15.77	15.77	15.82	Third Torsional (3T)
9	22.17	22.17	22.17	22.17	22.17	15.53	15.53	14.84	15.58	15.58	22.17	22.17	22.17	22.17	22.17	Third E/W Mode (3EW)
10	25.39	25.39	25.39	25.39	25.39	23.10	23.10	23.10	23.10	23.10	25.15	25.15	25.15	25.15	25.15	Fourth E/W Mode (4EW)

<sup>2</sup> Natural frequencies not possible to determine with confidence for this mode shape.

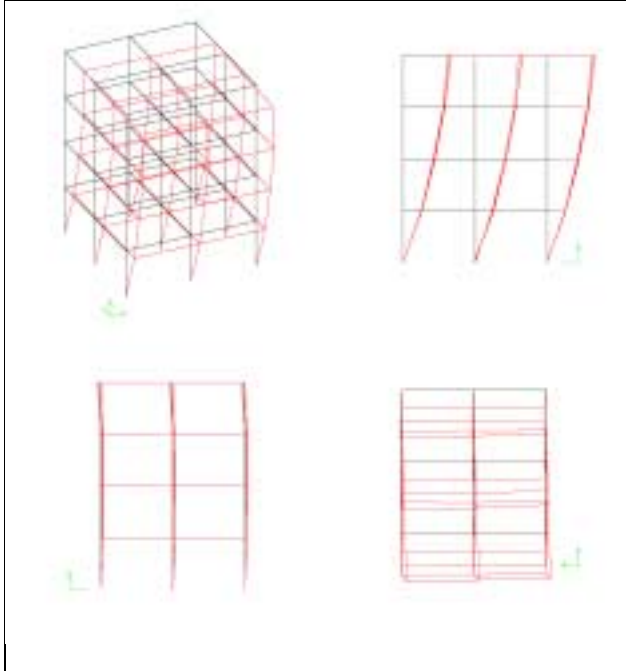


Figure 4 View of 1<sup>st</sup> mode for case I to V

#### FINITE ELEMENT MODEL AS BENCHMARK

A finite element model of the frame was created using the FEMtools<sup>®</sup> program with the aim to generate a benchmark for the steel frame. The structure is modelled as a beam-column element frame. Since the beam-column connections do not have 100% rigid link characteristics, the connections of the frame are modelled based on the paper published by Ventura et al., 1997 Ref. [4]. For the braces used as axial elements in the finite element model, the moment of inertia was assumed zero and only cross-section area was considered. To model the mass in the finite element model, a lumped concentrated mass approach was used.

The finite element model was analysed for natural frequencies. The obtained results indicated difference with natural frequencies of the test data. The difference in natural frequency of the analysed finite element model and the experimental results can be attributed to various parameters, such as inaccurate modelling, construction error, weak connection, error in mass distribution and pre-stressed brace elements due to over tightening of their bolts.

To create a better matching finite element model, the preliminary finite element model was upgraded by correlation with the experimental results of case I and VI. The correlation was performed on selected parameters such as member and mass properties. The member properties defined as highly uncertain are such as moment of inertia (I), the cross-section area (A), Young's modulus of elasticity and the connection rigidity.

To evaluate the correlation of the prepared finite element model with the tested model, Modal Assurance Criteria (MAC), FRF Pairing and Shape pairings were utilized. Figure 5 shows the final MAC for the upgraded model. The resultant finite element is used as benchmark for comparison with the damaged tested model to detect the damage.

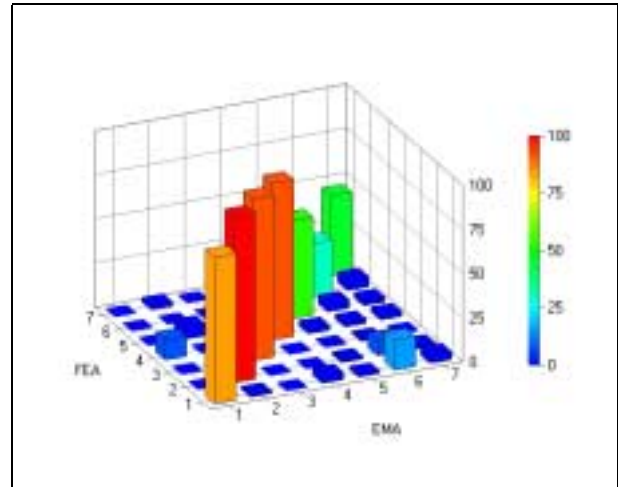


Fig. 5. Modal Assurance Criteria (MAC) for the benchmark finite element model.

#### DAMAGE DETECTION

To identify the damage, the finite element model was updated with the experimental results from the damaged cases and significant reduction in member properties were recorded as damage. In the process of updating the finite element model with the experimental results of the damaged cases, the following steps were applied.

**Parameter selection** – Based on the performed sensitivity analysis, the uncertainty of several different parameters was selected for the updating process. An example of a few of these parameters is the Young's modulus of elasticity of the steel, the moment of inertia for the beam and column members and the cross-section area of the braces.

The mass was not defined as an uncertain parameter in the updating process for damage detection, since any uncertainty was judged to be insignificant. In the case of member properties the uncertainty was much greater, so the parameters were allowed to change. By permitting independent variation of these parameters for different groups of structural elements it is possible to have an estimation of the sensitivity in the model to material and member properties and how these affect the overall dynamic behaviour of the structure.

The moment of inertia, and as a consequence, the total stiffness of the beam and column is one of the most uncertain parameters in the steel frame. The value of I is highly uncertain and is sensitive in the beam to column connections. The cross-section area for braces, A, is

another sensitive parameter to consider for the damage cases.

**Model update** – The objective of model updating is to adjust the values of selected parameters such as that a reference correlation coefficient is minimized. In sensitivity-based parameter estimation, the functional relationship between the modal characteristics and the structural parameters can be expressed in terms of a Taylor series expansion limited to the linear term, which can be written as (Ref. [5]):

$$\{R_e\} = \{R_a\} + [S] (\{P_u\} - \{P_o\}) \quad (1)$$

or

$$\{\Delta R\} = [S] \{\Delta P\} \quad (2)$$

where

- $\{R_e\}$  vector containing the reference system response (experimental data),
- $\{R_a\}$  vector containing the predicted system responses for a given state  $\{P_o\}$  of the parameter values,
- $\{P_u\}$  vector containing the updated parameter values and
- $[S]$  sensitivity matrix.

The correlation of the responses and the computation of MAC values were done at 14 points. (3 points per floor and 4 different level; plus 2 points in the base level). The correlation and updating of the FE model with the experimental results of the damaged structure was performed using seven modes.

**Damage evaluation and assessment** – The change in the parameters were closely investigated. The structural damage was assessed based on the member and material property reduction. The member properties in the enhanced finite element model were compared with the updated model of damaged cases.

Fig. 6 to 9 shows the updated model of the damaged cases. The members with the maximum reduction in properties are marked in these figures. In Fig. 6, the detected damage was located in the cross-sectional area of the east-west braces of the first storey. In Fig. 7, the detected damage was located in the cross-section area of the north-south braces of the second storey. However, while the damaged element was not determined, the approximate location and element type is in good agreement with the actual damage cases.

The damage shown in Figs 8 and 9 were detected in the beam and connection elements. Since the induced damage was insignificant, many parameters (such as column properties) were not permitted to change throughout the model updating of the benchmark. In figures 8 to 9 the damaged element was not allocated but the approximate location and element type is in good agreement with the actual damage cases V and VIII.

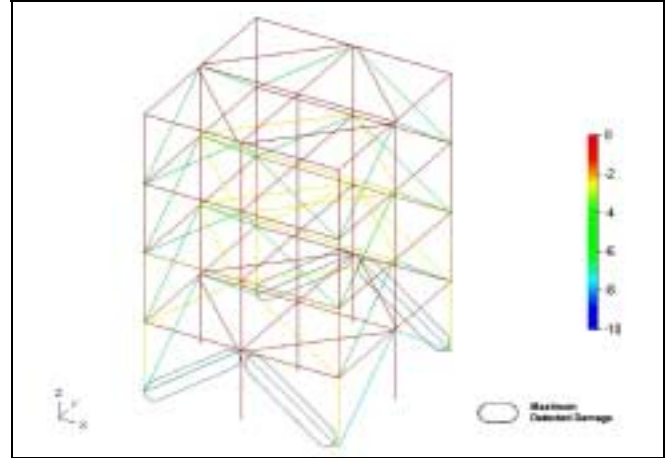


Fig. 6. Detected damage in for case III, determined in the ground storey and in the braces of the north side.

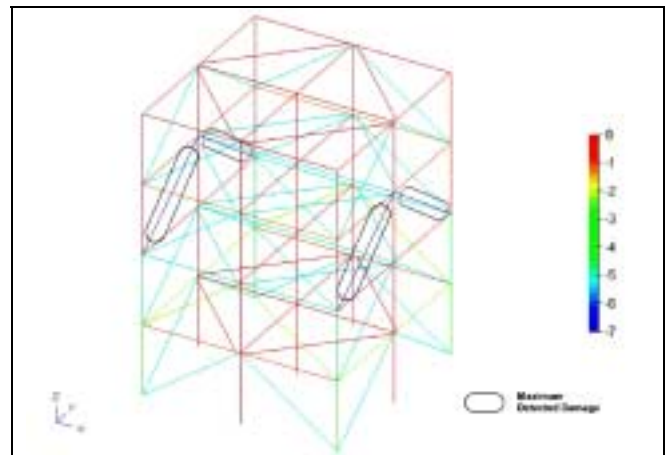


Fig. 7. Detected damage for case IV, determined in the second storey and in the braces of the west side.

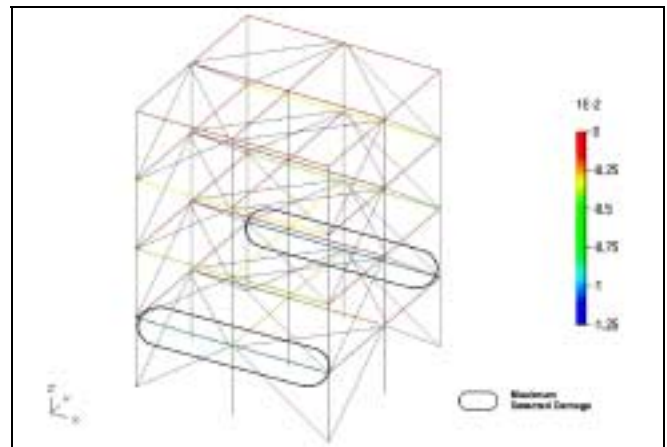


Fig. 8. Detected damage for case V, determined in the first floor and in the beam of the north side.

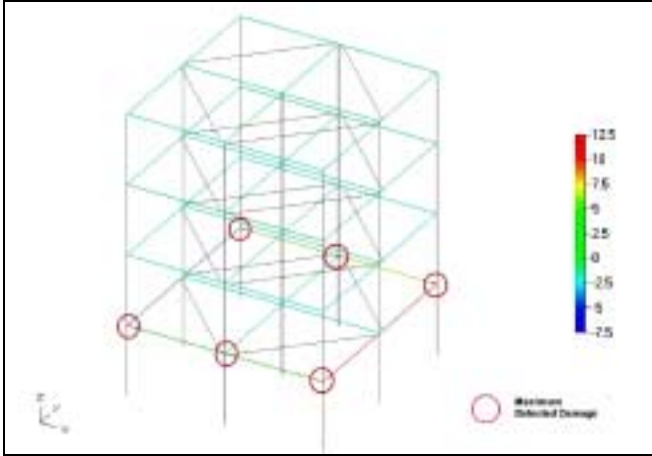


Fig. 9. Detected damage in case VIII, determined in the first floor and in the connections.

## CONCLUSIONS

The ambient vibration data analysis of the steel frame was sufficient to identify 9 modes for the cases I to V and 10 modes for the cases VI to X. The fundamental north-south frequency was identified as 4.688 - 4.395 Hz for cases I to V and 1.66 Hz for cases VI to X. The fundamental east west frequency was 4.980 - 4.883 Hz for cases I to V and 2.832 - 2.734 Hz for cases VI to X. The fundamental torsional mode was also observed uncoupled at 10.35 - 9.473 Hz for cases I to V and 3.320 Hz for cases VI to X.

The ambient vibration analysis showed clean, well defined modes, especially in the lower modes. When comparing the results from the testing done by forced vibration, high damping noise is observed. However a consistent correlation throughout the modes is present.

Of the five different damage cases investigated, four cases were successfully predicted. For the case where damage was not properly identified it was found that the induced damage was not significant enough to produce noticeable changes in the modal properties of the structure, and thus the sensitivity analyses were not able to provide a reliable identification of the presence of damage.

Also it was discovered that induced damage would affect the results significantly. Hence a higher agreement between results would have been reached if some specific parameters had been taken into consideration. These parameters are as follows.

1. **More Damage** - To incorporate the damage effect properly, more damaged cases should be created and with a higher severity.
2. **Type of Damage** - Introduced damage should be exercised in different parts of the structure. The massive steel plates in the ambient vibration test could introduce a very strong slab effect, therefore it is suggested that the created damage would have affected this element too.

3. **Asymmetry** - If asymmetry is to be introduced, enough loading has to be placed to create a considerable change in the torsional behaviour of the structure.

The damage detection was based on only seven modes. If more modes had been considered, the results would have been of better quality. To increase the accuracy of the obtained natural frequency, the above-mentioned issues have to be taken into account.

## ACKNOWLEDGMENTS

The author wish to acknowledge Drs. James L. Beck of Caltech University, Dennis Bernal of Northeastern University in Boston and Sheryl Dyke of Washington University in St. Louis, who provided valuable help in the testing activities and testing methodology. The measurements were conducted with the assistance of participating students, Suzy Hyun and Ellen Beckmann of Washington University, Human Ghalibafian of University of British Columbia and Kevin Yuen of Caltech Universities.

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