

# OMA testing by SLDV with FEM Pre and Post-test Analysis

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## NOMENCLATURE

$C$	Speed of light
$f$	Frequency
$\lambda$	Wavelength
$v$	Velocity in direction of laser beam
$G_{xx}(j\omega)$	Input Power Spectral Density matrix
$G_{yy}(j\omega)$	Output Power Spectral Density matrix
$H(j\omega)$	Frequency Response function (FRF) matrix
$m$	Total number of modes
$\lambda_k$	Pole of the $k^{\text{th}}$ mode
$\sigma_k$	Modal damping of the $k^{\text{th}}$ mode
$\omega_{dk}$	Damped natural frequency of the $k^{\text{th}}$ mode
$\zeta_k$	Critical damping of the $k^{\text{th}}$ mode
$\omega_{0k}$	Undamped natural frequency of the $k^{\text{th}}$ mode
$[R_k]$	Residue matrix of the $k^{\text{th}}$ mode
$\Psi_k$	Mode shape of the $k^{\text{th}}$ mode
$\gamma_k$	Modal participation of the $k^{\text{th}}$ mode
$[A_k]$	$k^{\text{th}}$ residue matrix of the matrix $[G_{yy}]$
$r_{0k}$	Initial value of the correlation function
$r_{pk}$	$p^{\text{th}}$ extrema of the correlation function
$\rho$	Density
$E$	Young's Modulus

## ABSTRACT

Operational Modal Analysis (OMA) is a technique for identification of modal parameters by measurement of only the system's response.

On many lightweight structures, such as load-speaker cones and disk drive read/write heads, is impossible or impractical to measure the input forces. Another characteristic of lightweight structure is their sensitivity to mass loading from sensors. The Scanning Laser Doppler Vibrometry (SLDV) allows response measurements to be taken without mass loading.

OMA test results can not be directly used for Structural Dynamics Modification (SDM) because the results are unscaled. Instead, the modal results from the OMA test are used to update a Finite Element Model (FEM). The updated FEM is then used to analytically predict the behavior of a modified system.

## **INTRODUCTION AND THEORY**

### ***Operational Modal Analysis***

Operational Modal Analysis, also known as Output Only Modal Analysis or Ambient Modal Analysis, has for over a decade been used for extracting modal parameters from civil engineering structures and is now also being used for mechanical structures in on-road and in-flight testing [1], [2].

The advantage of this method is that, generally, no artificial excitation needs to be applied to the structure. If artificial loading is required, the forces do not need to be measured. All parameter estimation is based on the response signals, thereby minimizing the amount of work required for test preparation.

As the loading forces are unknown in Operational Modal Analysis, specially designed modal parameter estimation techniques need to be used. In classical modal analysis, many validation tools are based on the known loading force, but as this is not known, other validation tools must be implemented. In addition, different estimation techniques are used, so that results can be compared. Using Operational Modal Analysis, the object operates under real operational conditions, which means that no shakers are needed and the boundary conditions during the test are those of the object under normal operating conditions. This facilitates the installation work and ensures that the measurements are performed under realistic operational conditions.

The set-up is as simple as that for Operational Deflection Shape measurements (ODS), but using Operational Modal Analysis we get the true modal parameters: natural frequency, damping and mode shape. This is unlike ODS where only the actual deflection pattern is observed.

The ability to get the real modal parameters is illustrated here by a measurement on a rectangular plate.

### ***Finite Element Updating***

To keep development time and cost competitive, industry relies on computerized simulation tools. Finite element analysis (FEA) is a powerful technique to simulate the behavior of a product under various types of loadings. The FEA method has matured over the past three decades to a point where design, meshing, analysis and post-processing are becoming highly integrated and automated. This predictive approach relies on the quality of the simulation model, the software to analyze it and the engineering judgment of the analyst interpreting the results.

In order to keep up with quality requirements, simulation models and procedures must be validated. A good way to do this is testing.

Experimental methods to support product design and analysis are based on prototype measurements under laboratory conditions or in real-life situations. They are effective to learn about the product, and the environmental conditions. In a competitive world, however, a trial-and-error design and analysis approach involving a series of prototypes is too time-consuming and expensive. It is therefore necessary to reduce the number of iterations on prototypes. This can be achieved by deriving more information on prototype testing and by shifting prototyping to later in the design process.

Integrating test and analysis enables synergistic processes from which the entire engineering team can benefit. Some examples:

Test results are used as reference data to validate and refine a finite element model (error localization, correlation analysis, model updating). Unknown or badly known physical properties can be identified and uncertainties in finite element models better assessed.

Hybrid models that contain partially FE models and partially test models can be developed to build more complete models that include all essential components while maintaining a good balance between model size and performance.

Discrepancies between FEA results and reference data, like test data, may be due to uncertainty in the governing physical relations (for example, modeling nonlinear behavior with the linear FEM theory), the use of inappropriate boundary conditions or element material and geometrical property assumptions and modeling using a too coarse mesh. These 'errors' are in practice rather due to lack of information than plain modeling errors. Their effects on the FEA results should be analyzed and improvements must usually be made to reduce the errors associated with the FE model. Model updating has become the popular name for using measured structural data to correct the errors in FE models.

Model updating works by modifying the mass, stiffness, and damping parameters of the FE model until an improved agreement between FEA data and test data is achieved. Unlike direct methods, producing a mathematical model capable of reproducing a given state, the goal of FE model updating is to achieve an improved match between model and test data by making physically meaningful changes to model parameters which correct inaccurate modeling assumptions.

### **Scanning Laser Vibrometry**

A Laser Doppler Vibrometer (LDV) is based on the frequency shift that a laser beam goes through when reflected off of a structure with a velocity in the direction of the beam.

A LDV has as its main component: a laser, typically Helium-Neon, which produces a laser beam with a stable wavelength,  $\lambda$ , (632.8 nanometers) and frequency,  $f$ , ( $4.74 \times 10^{14}$  Hz). The wavelength and frequency are related by the equation:

$$C = \lambda * f \quad (1)$$

Where C is the speed of light ( $2.9977 \times 10^8$  m/s).

This beam is typically optically split inside the LDV to produce two beams.

One beam, the reference beam, remains inside the instrument and acts as a frequency reference. The second, measurement beam, leaves the instrument.

The measurement beam reflects off of a point on the structure. In reflecting off the structure the measurement beam's frequency is shifted due to the Doppler effect. The frequency shift,  $\Delta f$ , is given by the formula:

$$\Delta f = \frac{2v}{\lambda} \quad (2)$$

Where v is the velocity of the structure in the direction of the laser beam and  $\lambda$  is the wavelength of the laser beam.

The reflected beam is back scattered to the LDV where it is recombined with the reference beam and the frequency shift,  $\Delta f$ , is measured by one of several methods.

The frequency shift is typically converted by the LDV into an analog voltage for measurement by standard instrumentation like oscilloscopes, Fast Fourier Transform (FFT) Analyzers or Analog-to-Digital (A/D) converters.

### **Advantages of Laser Doppler Vibrometry**

Using a laser beam to measure velocity has many practical advantages.

One of the chief advantages of the LDV is that it is non-contacting in reference to the structure under test. Only the laser beam encounters the structure and it has virtually no momentum and has no effect on all but the

smallest, microscopic structures.

The fact that a laser has virtually no mass loading effects give it a significant advantage over contacting sensors like accelerometers and strain gauges whose own mass changes the systems they are measuring.

The low power, less than 2 milliwatt, used in most commercially available measurement LDV does not heat the structure under test.

Non-contacting also means that only the laser beam is subject to the environment on the surface of the structure being measured. Since the laser beam is unaffected by many phenomena like high temperature, high radiation levels, or magnetic fields, a LDV can be used on structures with harsh environments where contacting sensors could not function or would need cooling or shielding to operate.

Even on structures which are not sensitive to mass loading or which do not have harsh environments, the LDV has another advantage over contacting techniques like accelerometers. By its fundamental design, a LDV uses a laser beam to transmit the velocity information. This is of benefit on portions of the structure that are rotating relative to the rest of the structure.

The practical difficulty with contacting sensors and rotating components is getting the signal from rotating sensors to the non-rotating data acquisition equipment via a slip-ring or telemetry.

Even on structures without rotating components, the fact that the laser beam can measure over long distances, from tens to hundreds of meters depending on the power of the laser and the reflectivity of the object under test, gives the LDV an advantage over contacting measurement techniques that require wires to be run to each measurement point or the use of telemetry.

A further difficulty when measuring on large structures is gaining physical access to each measurement point to mount a contacting sensor like an accelerometer.

#### Scanning Laser Doppler Vibrometer (SLDV)

Even on structures where it is relatively easy to access and mount contacting sensors, the Laser Doppler Vibrometer (LDV), and particularly the Scanning Laser Doppler Vibrometer (SLDV) offer reduction in test setup time and sometimes even data acquisition time on test involving many physical points or Degrees of Freedom (DOF).

The SLDV is the same as the LDV with the addition of two mirrors which allow the laser spot location to be moved horizontally and vertically on a test object. Using mirrors to move the laser is faster and usually more repeatable than manually or mechanically repositioning a LDV on a tripod or other mount.

A SLDV can be used to position the laser spot at one point, take a measurement at that point, and then quickly reposition the beam (typically less than 10 milliseconds) for the next measurement.

Even with this fast scan speed, the SLDV is still, fundamentally a single point measurement device. A line scan technique has been applied to SLDV but even with these techniques the SLDV requires stationarity in the system under measurement [3].

The SLDV will reduce the test time on multiple DOF tests by not requiring the mounting of a transducer at every measurement point which can be a significant savings in setup time and test management.

Compared to a test setup with one or more shakers, accelerometers at each measurement DOF and the accompanying large channel count data acquisition systems, a SLDV test takes longer to acquire the data with the related assumption that the system is stationary throughout the laser scan. But compared to a roving test setup with a hammer, one or more reference accelerometers and less expensive, low channel count data

acquisition system, a SLDV test is faster in acquiring the data with same assumption that the system is stationary during the test for both the roving hammer and scanning laser techniques.

One advantage that the SLDV test technique has compared to the simultaneous accelerometer technique is its ability to achieve greater spatial resolution. Accelerometers have a spatial resolution limit that is their physical size. An accelerometer averages the motion of the area covered by its base. The laser spot of a LDV or a SLDV also averages the motion covered by its spot. The spot diameter of the LDV and SLDV is a function of the instruments optics and the distance to the object but typical values are 0.3 mm at 5 m and 1.3 mm at 20 mm. Spot diameters for LDV's can be significantly smaller when measuring through the optics of a microscope.

The final advantage that the LDV and SLDV have due to their non-contacting nature is the fact they are not present on the object. This is a benefit on objects in wind and water tunnels where the presence of transducers and cabling in the flow field can change the measurement [4].

Because the laser beam of the LDV is stream of photon and not a mechanical structure like an accelerometer, the LDV can more easily measure to higher frequencies. The accelerometer is often limited by its mounting frequency. This frequency is set by the accelerometer's own mass and the stiffness of its connection to the structure. Even for small accelerometers with very stiff mounting, there are internal resonances within the crystal/mass sensing structure. Because of the high amplifications and low damping at a piezo-electric (PE) accelerometer's resonant frequency, PE accelerometers are limited to approximately 20% of their resonant frequency.

See reference [5] for a further discussion on LDV measurements.

#### Limitations

There are several reasons why accelerometers are more commonly used for vibration measurements than LDV or SLDVs.

The first significant limitation is that the laser beam needs to have a clear line of sight from the LDV to the point to be measured. The laser can be reflected by mirrors, like the positioning mirrors used in a SLDV, but there still needs to be a clear path from the front of the LDV to one or more external mirrors and finally to the measurement point. This can be difficult to achieve on complex structures like an engine in a vehicle or satellites with multiple antennae and other appendages.

Line of sight also becomes an issue when making measurements into a test chamber. The laser beam can pass through the glass of observation ports but there is loss of energy each time the beam passes through the glass. This loss of energy can reduce the maximum measurement distance or require a more reflective target.

The LDV and SLDV require that the structure, at the point of measurement, reflects like back to the LDV. This is a requirement because the frequency shift, related to the structures velocity, is only present in the measurement beam after it is reflected.

Modern LDVs are sufficiently sensitive to measure the reflection from most surfaces. Measurements are commonly made on difficult surfaces like rotating black tires and speaker cones without difficulty. Factors like maximum stand off distance, focus quality and focus depth become more important as the reflectivity of the surface decreases.

Specular reflective surfaces like mirrors are actually very difficult to measure with a LDV. The problem with these surfaces is that almost none of the energy is back scattered to the LDV. It is common practice to dull these surfaces with talcum powder or developer's spray to improve the back scatter. This is not always acceptable and limits LDVs on applications with this type of reflectivity.

Even on surfaces with good back scatter like sand casted metal surface, LDV may have difficulty measuring. This is because the rough surface cause points of low energy in the back scattered beam. A common technique to

overcome this is to slightly move or “dither” the laser spot location to find a point with higher energy. A very small change in laser spot position can have a large effect on the strength of the back scattered beam.

The ability of SLDVs to quickly move the laser spot makes it simpler to implement dithering in SLDVs.

An LDV can only measure velocity in the direction of the laser beam. This limits the amount of information they can give about the motion at a point.

Because the amount of back scattered light from an object is inversely proportional to the angle of incidence, it is very difficult to make in plane measurements with an LDV.

There are other techniques which give more than one DOF at a measurement location but they have their own limitations.

3D Vibrometers have a limited tolerance around their nominal stand-off distance (typically 1-3 mm for nominal stand-off distances of 100-300 mm). This means that the sensing head needs to be kept a constant distance from each measurement point. This requirement causes the sensing head (or object) to be moved from measurement point to measurement point which increases test time. 3D Vibrometers still has the same requirements for line of sight and reflectivity as LDVs.

A LDV can be combined with a two-dimensional position sensitive detector to measure the velocity of a point plus the point’s two out of plane rotations [6]. It requires a LDV and additional instrumentation, restricts the working distance and requires a very reflective surface. It has been applied to measurements on hard disk drive components.

It is possible to combine measurements from either the same LDV from multiple measurement locations or multiple LDVs viewing the same point simultaneously from different directions.

Both techniques require an accurate dimensional model of the object and the relative location of LDV for each measurement in order to accurately combine the measurements.

Both multiple measurements with the same LDV or simultaneous measurements with multiple LDVs have the standard LDV difficulties with line of sight and difficulty measuring in-plane motion.

Finally, using multiple LDVs magnifies the most significant drawback that LDVs have compared to other measurement technologies, the cost. LDV are complicated instruments with laser tubes, optical lenses, laser detectors and frequency demodulators. These components come at significant expense with corresponding increase in complexity.

## **Test Procedure**

### ***OMA Test on operating system***

The specimen used is a rectangular plate (29cm×25cm×0,7cm), resting on a foam pad. The plate was excited by an electric motor at one corner of it. The measurements were taken using a Scanning Laser Doppler Vibrometer to avoid mass loading, on 36 DOF’s (Degrees of Freedom). 4 accelerometers were placed on the plate as references transducers, as seen in figure 1.

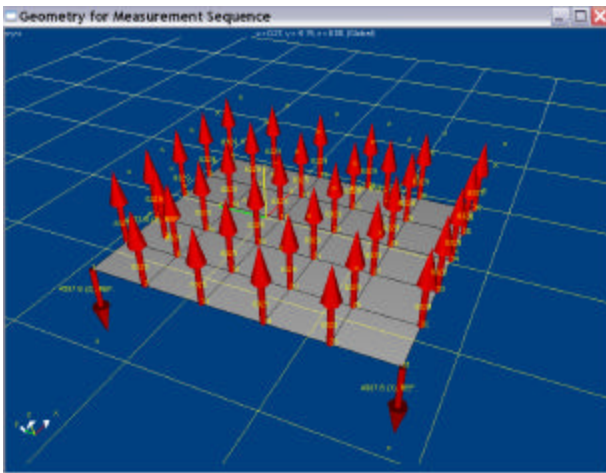
The data acquisition system used was a portable PULSE™ analysis platform, equipped with a Data Acquisition interface for Modal Testing.



**Figure 1: Back, Front, and Side views of test setup showing test object and SLDV**

### **Data Acquisition**

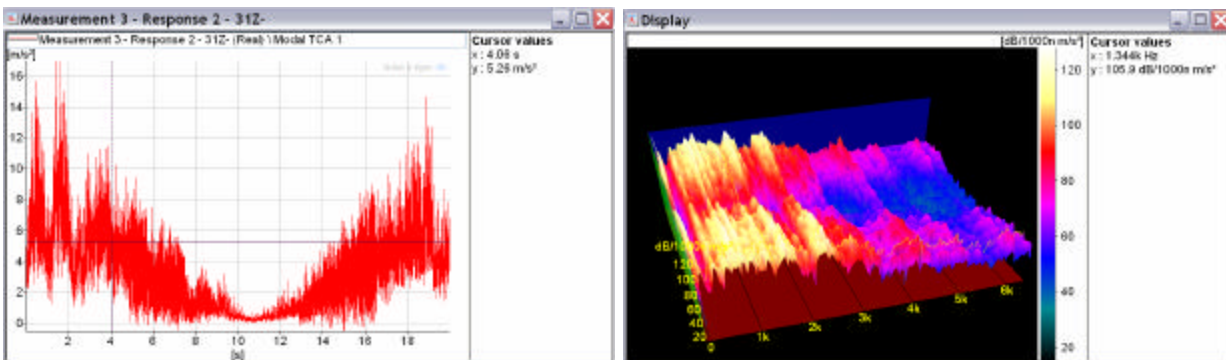
The PULSE Modal Test Consultant™ was used to set-up the hardware, create the geometry, and assign the measurements to the Degrees of Freedom. The reference accelerometers were kept at 4 well-chosen points on the plate, and all measurements were performed on the 36 DOF's with the laser beam.



**Figure 2: Geometry and measured Degree of Freedom**

The raw time data was captured by a “Time Capture Analyzer” for each measurement set. The capture analyzer was setup for a frequency span of 6.4 kHz, and a track length of 20 seconds, to allow for a run-up, and then run-down of the electric motor.

A capture of a response time signal at a Degree of Freedom can be seen here.



### Figure 3: Capture of a response time signal at STFT at a Degree of Freedom

A Short Time Fourier Transform (STFT) analysis provides a Time-Frequency representation of all the responses captured. The STFT exhibits already the spectral components related to the structural resonances, as constant frequency lines in the contour plot.

#### Modal Extraction & Result

Preliminary signal processing

The first step of the analysis is to perform a Discrete Fourier Transform (DFT) on the raw time data, to obtain the Power Spectral Density Matrices that will contain all the frequency information.

Power Spectral Density matrices are then estimated from to a Singular Value Decomposition, and a modal extraction is then performed using Frequency Domain Decomposition techniques.

Frequency Domain Decomposition background

The Frequency Domain Decomposition (FDD) is an extension of the Basic Frequency Domain (BFD) technique, or more often called the Peak-Picking technique. This approach uses the fact that modes can be estimated from the spectral densities calculated, in the condition of a white noise input, and a lightly damped structure. It is a non-parametric technique that estimates the modal parameters directly from signal processing calculations.

The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the Spectral Density matrix. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the system for each singular value [7], [8].

The relationship between the input  $x(t)$ , and the output  $y(t)$  can be written in the following form:

$$[G_{yy}(j\omega)] = [H(j\omega)]^* [G_{xx}(j\omega)] [H(j\omega)]^T \quad (3)$$

where  $G_{xx}(j\omega)$  is the input Power Spectral Density matrix, that turns out to be constant in the case of a stationary zero mean white noise input. This constant will be called  $C$  in the rest of the mathematical derivation.  $G_{yy}(j\omega)$  is the output PSD matrix, and  $H(j\omega)$  is the Frequency Response function (FRF) matrix. As seen in equation (3), the output  $G_{yy}$  will be highly sensitive to the input constant  $C$ . The rest of the equation derivations and single degree of freedom identification will provide relevant results, only by assuming that the input is effectively represented by a constant value (mean Gaussian). It is therefore important to realize how this input assumption will be crucial to the technique.

The FRF matrix can be written in a typical partial fraction form (used in classical Modal analysis), in terms of poles and residues

$$[H(j\omega)] = \frac{[Y(\omega)]}{[X(\omega)]} = \sum_{k=1}^m \frac{[R_k]}{j\omega - \mathbf{I}_k} + \frac{[R_k]^*}{j\omega - \mathbf{I}_k^*} \quad (4)$$

with

$$\mathbf{I}_k = -\mathbf{s}_k + j\omega_{dk} \quad (5)$$

$m$  being the total number of modes,  $\lambda_k$  being the pole of the  $k^{\text{th}}$  mode,  $\sigma_k$  the modal damping and  $\omega_{dk}$  the damped natural frequency of the  $k^{\text{th}}$  mode:



$$\mathbf{w}_{dk} = \mathbf{w}_{0k} \sqrt{1 - \mathbf{V}_k^2} \quad (6)$$

$$\mathbf{V}_k = \frac{\mathbf{s}_k}{\mathbf{w}_{0k}} \quad (7)$$

with

$\zeta_k$  being the critical damping and  $\omega_{0k}$  the undamped natural frequency, both for the mode  $k$ .

$[\mathbf{R}_k]$  is called the residue matrix and is expressed in an outer product form:

$$[\mathbf{R}_k] = \mathbf{y}_k \mathbf{g}_k^T \quad (8)$$

where  $\Psi_k$  is the mode shape,  $\gamma_k$  the modal participation vector. All those parameters are specified for the  $k^{\text{th}}$  mode.

The transfer function matrix  $[\mathbf{H}]$  is symmetric, and an element  $H_{pq}(j\omega)$  of this matrix is then written in terms of the component  $r_{kpq}(j\omega)$  of the residue matrix as follows:

$$H_{pq}(j\omega) = \sum_{k=1}^m \frac{r_{k(p,q)}}{j\omega - \mathbf{I}_k} + \frac{r_{k(p,q)}^*}{j\omega - \mathbf{I}_k^*} \quad (9)$$

Using the expression (3) for the matrix  $G_{yy}$ , and the Heaviside partial fraction theorem for polynomial expansions, we obtain the following expression for the matrix output PSD matrix  $\mathbf{G}$ :

$$[\mathbf{G}_{yy}(j\omega)] = \sum_{k=1}^m \frac{[\mathbf{A}_k]}{j\omega - \mathbf{I}_k} + \frac{[\mathbf{A}_k]^*}{j\omega - \mathbf{I}_k^*} + \frac{[\mathbf{B}_k]}{-j\omega - \mathbf{I}_k} + \frac{[\mathbf{B}_k]^*}{-j\omega - \mathbf{I}_k^*} \quad (10)$$

where  $[\mathbf{A}_k]$  is the  $k^{\text{th}}$  residue matrix of the matrix  $[\mathbf{G}_{yy}]$ . The matrix  $\mathbf{G}_{xx}$  is assumed to be a constant value  $\mathbf{C}$ , since the excitations signals are assumed to be uncorrelated zero mean white noise in all the measured DOF's. This matrix is Hermitian and is described in the form:

$$[\mathbf{A}_k] = [\mathbf{R}_k] \mathbf{C} \sum_{s=1}^m \frac{[\mathbf{R}_s]^H}{-\mathbf{I}_k - \mathbf{I}_s^*} + \frac{[\mathbf{R}_s]^T}{-\mathbf{I}_k - \mathbf{I}_s} \quad (11)$$

The contribution of the residue has the following expression:

$$[\mathbf{A}_k] = \frac{[\mathbf{R}_k] \mathbf{C} [\mathbf{R}_k]^H}{2\mathbf{s}_k} \quad (12)$$

Considering a light damping model, we have the following relationship:

$$\begin{aligned} \lim_{\text{damping} \rightarrow \text{light}} [\mathbf{A}_k] &= [\mathbf{R}_k] \mathbf{C} [\mathbf{R}_k]^T = \mathbf{y}_k \mathbf{g}_k^T \mathbf{C} \mathbf{g}_k \mathbf{y}_k^T \\ &= d_k \mathbf{y}_k \mathbf{y}_k^T \end{aligned} \quad (13)$$

Where  $d_k$  is a scalar constant.

The contribution of the modes at a particular frequency is limited to a finite number (usually 1 or 2). The response spectral density matrix can then be written as the following final form:

$$[G_{yy}(j\omega)] = \sum_{k \in \text{Sub}(\omega)} \frac{d_k \mathbf{y}_k \mathbf{y}_k^H}{j\omega - \mathbf{I}_k} + \frac{d_k^* \mathbf{y}_k^* \mathbf{y}_k^{*H}}{j\omega - \mathbf{I}_k^*} \quad (14)$$

where  $\text{Sub}(\omega)$  is the set of modes that contribute at the particular frequency.

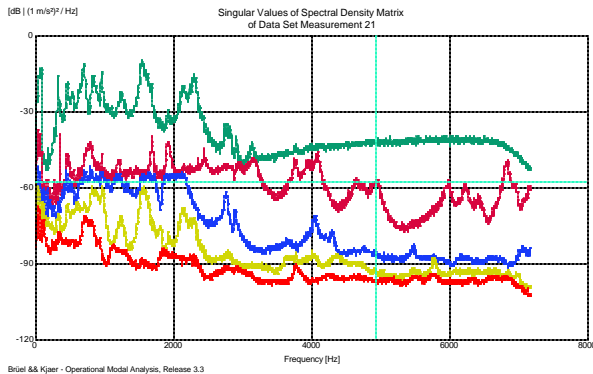
This final form of the matrix is then decomposed into a set of singular values, and singular vectors, using the SVD technique (Singular Value Decomposition). This decomposition is performed to identify Single Degree of Freedom Models to the problem.

### Singular Value Decomposition

This singular value decomposition is performed for each of the matrices at each frequency, and for each measurement. The spectral density matrix is then approximated after SVD decomposition.

The singular vectors correspond to an estimation of the Mode Shapes, and the corresponding singular values are the Spectral Densities of the SDOF system.

Figure 4 shows the result of the Singular Value Decomposition of the Spectral Density matrix, at the measurement 21. We obtain 5 singular values, and 5 singular vectors for each of the Spectral Density matrices. The singular values and their corresponding singular vectors are ranked in singular value descending order for each of the spectral density matrices, meaning that the first singular value will be the largest.

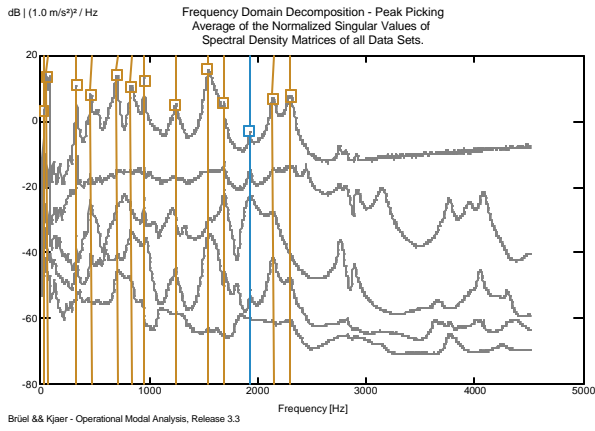


**Figure 4: Result of the SVD for the measurement 21**

This technique allows the identification of possible coupled modes that are often indiscernible as they appear on the Spectral Density Functions. If only one mode is dominating at a particular frequency, then only one singular value will be dominating at this frequency. In the case of close or repeated modes, there will be as many dominating singular values as there are close or repeated modes.

### Peak-Picking and Modal determination

Each of the SDOF systems obtained by the Singular Value Decomposition, allows us to identify the natural frequency, and mode shape (unscaled), at a particular peak.



**Figure 5: Peak Pick from SVD for the measurement 21**

Peak-Picking on the average of the normalized singular values of the PSD matrix for all data sets.

8 structural modes were extracted on the plate to perform the Finite Element Model updating.

Damping estimation

The Enhanced FDD technique allows extracting the resonance frequency and the damping of a particular mode by computing the auto, and cross-correlation functions. The SDOF Power Spectral Density function identified around a peak of resonance, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The resonance frequency is obtained by determining the zero crossing times, and the damping by the logarithmic decrement of the corresponding SDOF normalized auto correlation function.

The free-decay time domain function (that is the correlation function of the SDOF system) is used to estimate the damping for the mode  $k$ :

$$\mathbf{d}_k = \frac{2}{p} \ln \left( \frac{r_{0k}}{|r_{pk}|} \right) \quad (15)$$

where  $r_{0k}$  is the initial value of the correlation function, and  $r_{pk}$  the  $p^{\text{th}}$  extrema. The critical damping ratio for the mode  $k$ , is obtained by the formula:

$$\mathbf{V}_k = \frac{\mathbf{d}_k}{\sqrt{\mathbf{d}_k^2 + 4p^2}} \quad (16)$$

The damped natural frequency is obtained by linear regression on the crossing times corresponding to the extrema of the correlation function. The undamped natural frequency for the mode  $k$  is then:

$$f_{0k} = \frac{f_{dk}}{\sqrt{1 - \mathbf{V}_k^2}} \quad (17)$$

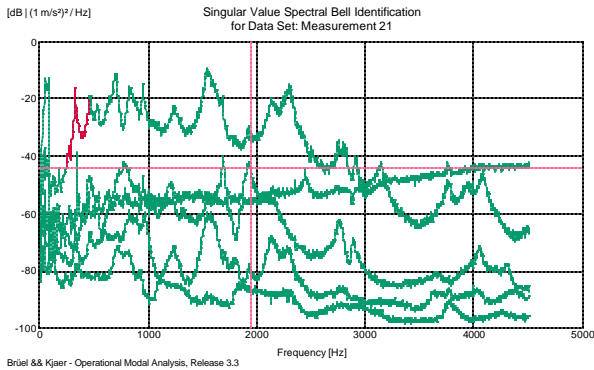
Both parameters and an improved version of the mode shapes are estimated from the **SDOF Bell functions**. The SDOF Bell function is estimated using the mode determined by the previous FDD peak-picking. The latter being used as a reference vector in a correlation analysis based on the Modal Assurance Criteria (MAC). A MAC value

is computed between the reference FDD vector and a singular vector for a particular frequency region. The MAC value describes the degree of correlation between 2 modes (it takes a value between 0 and 1) and is defined as follows for 2 vectors  $\Phi$  and  $\Psi$ :

$$MAC(\{\Phi\}, \{\Psi\}) = \frac{\|\{\Phi\}^* \{\Psi\}\|^2}{\|\{\Phi\}\| \cdot \|\{\Psi\}\|} \quad (18)$$

If the largest MAC value of this vector is above a user-specified **MAC Rejection Level**, the corresponding singular value is included in the description of the SDOF Spectral Bell function. The lower this MAC Rejection Level is, the larger the number of singular values included in the identification of the SDOF Bell function will be. An average value of the singular vector (weighted by the singular values) is then obtained.

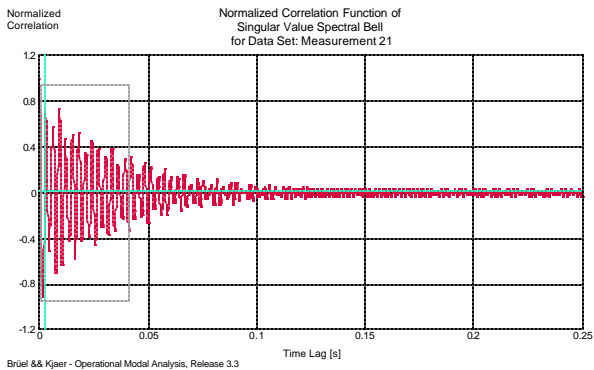
Figure 6 shows the estimated SDOF Bell function with a MAC rejection level of 0.8.



**Figure 6: Singular Value Spectral Bell identification for measurement 21**

The value of the MAC rejection criteria has to be chosen so that we obtain a good representation of the Bell function around the peak chosen, and not include any noise around it, often present in an Ambient Modal analysis.

Using this SDOF Bell function, we perform an inverse Fourier Transform for the determination of the damping and the natural frequency. The obtained normalized correlation function is shown in Figure 7.

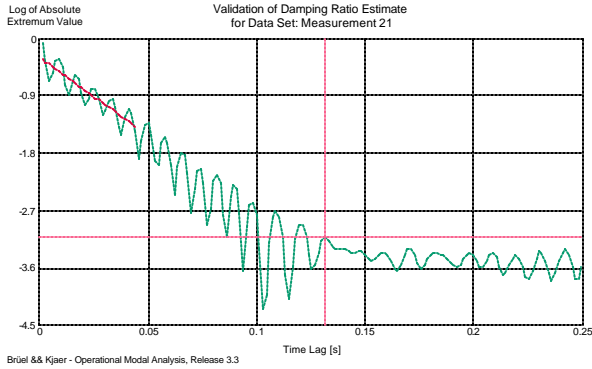


**Figure 7. Normalized Correlation function for measurement 21**

Figure 8 exhibits a typical response of a resonating system that decays exponentially. The scattered region indicates the part of the correlation function that is used for the estimation algorithm. In that particular example,

the modes are well spaced in the frequency domain, and will provide leakage-free correlation functions. In cases where frequency peaks are not clearly spaced, leakage will definitely affect the Inverse Fourier process, since only a limited frequency range is used for the Fourier calculations.

The damping is estimated by the logarithmic decrement technique from the logarithmic envelope of the correlation function. The estimation is performed by using a linear regression technique (red part of the curve in Figure 8).



**Figure 8: Damping Ratio estimation from the decay curve of the correlation function**

The resonance frequency is simply obtained by counting the number of times the correlation function crosses the zero axis.

The result of the regression is shown as the red line. It is important to note that the estimation of the damping and natural frequency is performed for each set of measurement. The result is then obtained by averaging all the data sets together. Both the average value and the standard deviation of the damping and natural frequency are calculated from the datasets.

### Creating initial FEM

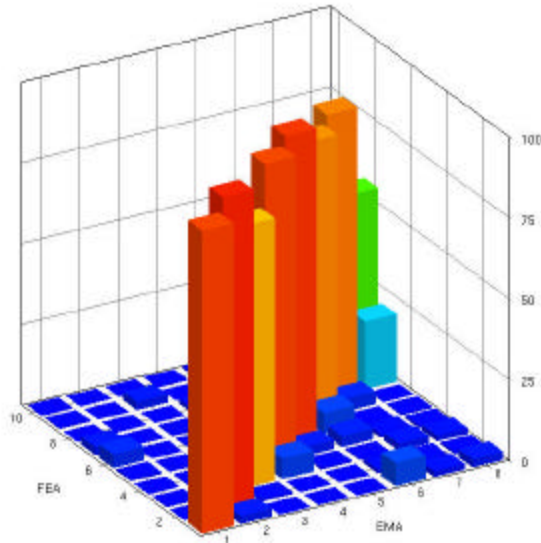
The FE model was created using quad 4 plate elements. The exciter was modeled as a mass element of 0.530 kg. It was assumed that the plate material is aluminum with following properties: a mass density of 2770 kg/m<sup>3</sup>, Youngs Modulus of  $7.31 \cdot 10^{10}$  Pa and a Poisson coefficient of 0.3.

An initial numerical modal analysis using the software's internal Lanczos solver followed by a geometrical correlation and mode shape correlation analysis, resulted in the following mode shape pairing table:

Pair #	FEA	Hz	EMA	Hz	EMA Damping (%)	Diff.	MAC
1	1	427.55	1	330.81	1.2	29.24	93.7
2	2	587.25	2	463.5	2.9	26.7	96.9
3	3	880.55	3	704.06	2.1	25.07	81.3
4	4	1044.11	4	837.52	2.8	24.67	93.4
5	5	1208.04	5	954.52	0.8	26.56	95
6	6	1553.53	6	1249.73	2.4	24.31	86.1
7	7	1997.31	7	1549.24	1.9	28.92	87.7

**Table 1: Mode shape pairs before updating**

The corresponding MAC matrix is shown in figure 9.



**Figure 9: MAC matrix before updating**

*Update FEM with OMA results*

FEMtools updating software uses well-proven iterative, parametric, modal and FRF-based updating algorithms using sensitivity coefficients and weighting values (Bayesian estimation).

The model updating method uses the discrepancy between FEA results and test, and sensitivities to determine a change in the update parameters that will reduce the discrepancy. The FE model is then reformed using the new values of the update parameters, and the process is repeated until some convergence criteria, analyzed by means of correlation functions, is met.

The aim of this model updating is to improve the frequency correlation as well as the total mass of the FEA model. Therefore the responses for the analysis are the first 8 eigen frequencies of the test model as well as the model's total mass.

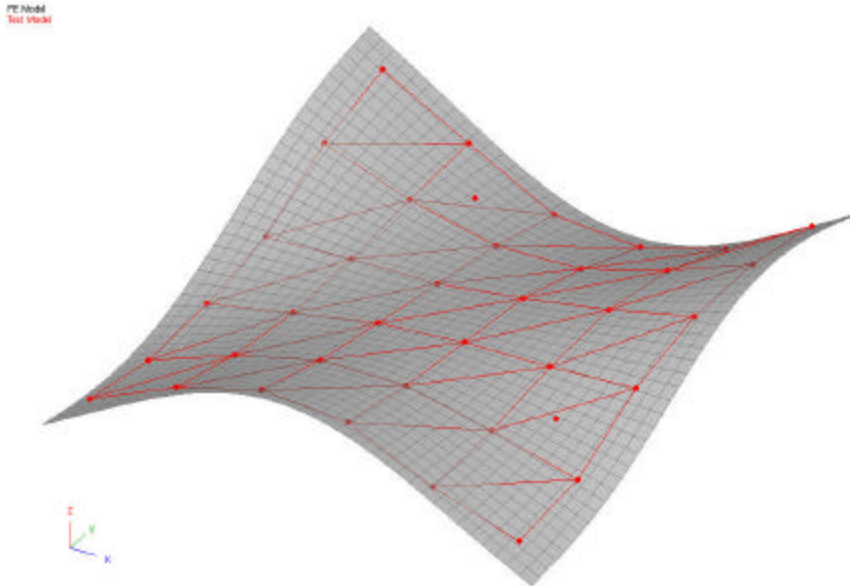
The updating parameters chosen are the material density and Young's modulus. These are used as global parameters, so that the values of these properties remain the same for all elements.

The model updating finally results in the following mode shape pairing table

Pair #	FEA	Hz	EMA	Hz	EMA Damping (%)	Diff.	MAC
1	1	337.36	1	330.81	1.2	1.98	93.9
2	2	464.93	2	463.5	2.9	0.31	98.1
3	3	701.04	3	704.06	2.1	-0.43	82.1
4	4	823.69	4	837.52	2.8	-1.65	93.3
5	5	960.54	5	954.52	0.8	0.63	96.1
6	6	1216.87	6	1249.73	2.4	-2.63	89.2
7	7	1583.94	7	1549.24	1.9	2.24	89.8
8	8	1682.55	8	1686.85	0.4	-0.26	56.4

**Table 2: Mode shape pairs after updating**

The frequency correlation has improved significantly to less than 3% for all mode shape pairs. Since only density and Young's modulus are used as global parameters and MAC was already good before updating, MAC improvements are very small. Figure 10 shows the fifth mode shape pair.



**Figure 10: Fifth mode shape pair after updating.**

*Applying a mass loading by using SDM to predict modified systems behaviour*

An extra mass loading is then applied to the rectangular plate and an OMA analysis is again performed.

To simulate the effect of adding this mass to the structure in the FEA model, a SDM technique is used. Structural Dynamics Modification (SDM) is a modal domain method to rapidly estimate the influence of many structural changes on the modal parameters. Because only a model geometry and modal parameters are used, SDM works as well on finite element data, test data and hybrid models. The structural changes can be modeled as simple springs, masses, dampers, or any type of finite element (bar, beam, shell, volume).

In the updated FE model a Modification Element of type mass of 0.123 kg is introduced and the FEA model is recalculated. Table 3 summarizes the results.

<b>Mode</b>	<b>Measured frequencies before adding a mass</b>	<b>Measured frequencies after adding a mass</b>	<b>Calculated frequencies of the updated FEA with extra mass</b>
<b>1</b>	330.81	319.909	327.681
<b>2</b>	463.499	440.463	453.85
<b>3</b>	704.059	673.829	699.393
<b>4</b>	837.525	805.005	809.354
<b>5</b>	954.516	894.952	955.126
<b>6</b>	1249.735	1191.08	1206.9
<b>7</b>	1549.244	1692.144	1583.927
<b>8</b>	1686.85	1773.126	1664.89

**Table 3: Eigen frequencies of the plate before and after adding a mass loading and calculated FEA frequencies.**

The corresponding mode shape pairing table is shown hereafter:

Pair #	FEA	Hz	EMA	Hz	Diff.	MAC
1	1	327.68	1	319.91	2.43	97.4
2	2	453.85	2	440.46	3.04	93
3	3	699.39	3	673.83	3.79	83.3
4	4	809.35	4	805	0.54	85.4
5	5	955.13	5	894.95	6.72	83.1
6	6	1206.9	6	1191.08	1.33	68.4

**Table 4: Mode Shape Pairs after adding a mass.**

## CONCLUSIONS & FUTURE WORK

A Scanning Laser Doppler Vibrometer (SLDV), combined with accelerometers as references, has been used to perform an Operational Modal Analysis measurement on a black plate excited by small electrical motor. The mode shapes match previous results obtained by traditional input-output modal analysis and OMA testing using only accelerometers. The damping at some resonant frequencies is higher than previously measured under free-free boundary conditions. We believe the higher than expected damping values are due to the non free-free boundary conditions.

Being able to take OMA measurements with a SLDV allows OMA tests to be performed on light-weight or hot structures. Now that OMA measurements by SLDV have been validated on a simple structure, future work would include measurements on more complex, real-world structures like speaker cones and hard-disk read/write arms.

The second part of the test showed that the OMA results could be used to update a Finite Element Model (FEM). This updated FEM was then used to predict the behavior of the original object plus an added mass. The results of the updated FEM model closely matched the results from a second OMA test of the original structure with a mass attached. These results show that OMA models can be used to update FEM and improve FEM capability to analytically predict the behavior of modified systems.

Using FEMs updated by OMA to predict a modified system's performance is important because the OMA results alone, can not be directly used for Structural Dynamic Modification studies because they are unscaled.

Future work would use the FEM for pre-test analysis to select the best point for reference and roving measurement locations. Pre-test analysis would help to alleviate the two main limitations of SLDV testing: line of sight and only uni-directional information at each measurement point. Pre-test analysis using FEM could also benefit OMA testing by reducing the number of roved measurement points which is directly related to the testing time.

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