Effect of Limited DOFs and Noise in Structural Updating

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ABSTRACT

In the field of structural updating, several methods have been introduced by many authors. Most of them attempt first to evaluate differences betweeen the numerical model and the actual experimental structure, then to compute a new updated numerical model by the use of the previous results. Those methods are all based on the idea that the dynamic behaviour of a structure is fully represented by dynamic parameters such as eigenvalues, eigenvectors and FRFs. From a theoretical point of view, all of the proposed methods have shown paramount capabilities to update the numerical representation of a structure, with respect to the number of mismodelled regions and with respect to the gap between the numerical and experimental model. Indeed, if the noise and actual experimental degrees of freedom are introduced, a lack of efficiency occours leading to incorrect results and sometimes the experimental identification of the structure, with conseguent numerical updating, is not possible. Furthermore, some methods, when dealing with experimental data, are able to identify structural modifications only if those induce quite big variations of the dynamic parameters while other methods show opposite behaviour. In this paper the effect of limited DOFs available from experimental tests and the effect of noise are both analyzed and results gained from the *Minimum Rank Perturbation Method*, from *Stubbs* sensitivity-based method and from *Predictor–Corrector* method used in *FEMTools* commercial code for updating are compared. Numerical tests have been performed on structural elements via F.E. analysis.

NOMENCLATURE

$[B]$	minimum rank perturbation matrix
$[H(\omega_k)]$	frequency responce function evaluated at ω_k
$[K]\,,[M]$	stiffness and mass matrix
$[S_p]$	perturbation matrix
$[\Delta K]$	stiffness error matrix
$[\Phi]$	eigenvector matrix
$\{\varepsilon\}$	error vector
$\{\Delta p\}$	updating coefficient vector
N	total number of degrees of freedom
ω	natural eigenfrequency
α_j	j^{th} structural parameter modification
$\chi_s,\,\chi_a$	shape and amplitude correlation functions

subscripts

$a,\,x$	analytical (numerical), and experimental
k	k^{th} frequency point
$r,\,t$	retained and truncated DOF
$u,\,0$	updated and initial

1 INTRODUCTION

The finite element method has been used for many years to predict the behaviour of mechanical components taking advantage from refinements in structural computations. Correspondingly, experimental modal analysis techniques have been improved by developments gained both in hardware and software. Although both structural characterizations, numerical and experimental, could be considered of high accuracy with respect to those available in the past years, limits of both numerical and experimental analysis arise if correlation between them is considered [1−3]. The need to better understanding the mechanical component, requesting an higher and higher level of correlation between models, is a relevant issue for industrials: to date only a comparison of the computational and measurements results has been performed, while the benefits that the model updating is capable to introduce are simply not considered. The main limiting issues for a common use of updating the techniques relate to the techniques themself that fail to give usefull updating informations and require an high level of experience of the structural analyst. In the updating procedures, the finite element representation and the number of experimental informations actually represents the weak elements. While the former still need to be improved to take into account, for example, non–linear elasto–dynamic behaviour, the experimental data gives a picture of the real structural component only with a limited set of data. Furthermore, from a numerical point of view, the selection of the structural parameters to be updated still represent a crucial choice that the analyst must face with. In this paper different updating methods have been considered and the results from numerical tests are reported. Specifically, the effect of the limited number of degrees of freedom, DOFs, incompleteness of the modal data, and noise in three different updating methods, *i.e.*, Minimum Rank Perturbation *MRP*, sensitivity based method from *Stubbs* and the Predictor Corrector *P-C* used in the commercial updating code *FEMtools*, have been investigated. Although the first two methods were initially intended for structural damage identification, they are however considered as updating schemes assuming that only slight differences between the dynamic characteristics of the numerical and experimental model are present. Finally a hybrid updating procedure capable to overcome limitations imposed to the mentioned methods by limited number of DOFs and incompleteness, has been proposed.

2 DESCRIPTION OF UPDATING METHODS

The updating methods considered in this paper belong to those classified as iterative sensitivity methods in which the main objective is to evaluate first the sensitivity matrix $[S_p]$ and then the values of such coefficients, $\{\Delta p\}$ which best fit the difference between the predicted, numerical, and measured, experimental, dynamic characteristics $\{\varepsilon\}$. The sensitivity matrix $[S_p]$ should be capable to take into account the effects on the dynamic behaviour of the structure induced by a unit variation of the parameter p, while the vector $\{\varepsilon\}$ represents the actual differences between eigenfrequencies, eigenmodes, and frequency response functions that dynamically characterize the structure. The resulting governing equation to be solved iteratively with respect to $\{\Delta p\}$ is:

$$[S_p]\{\Delta p\} = \{\varepsilon\} \tag{1}$$

A brief description of the updating methods analyzed is here outlined. Dealing with the Minimum Rank Perturbation method [4], $\{\varepsilon\}$ represents the projection onto the measured mode shapes of the numerical matrices obtained by a finite element discretization. Considering the undamped equation of motion, for each $i-th$ mode, it is possible to write:

$$\{\varepsilon\}_i = \left(-\omega_{xi}^2 [M] + [K]\right)\{\Phi\}_{xi} \tag{2}$$

where subscript x stands for experimental quantities. Evaluating $\{\varepsilon\}_i$ for each eigen–mode, the most sensitive mode to the differences between numerical and experimental structure and the most sensitive degree of freedom, could be achieved. The minimum rank perturbation matrix *i.e.*, $[B]$, is obtained minimizing the rank of $[S_p]$ following the idea that the modifications are present only in a few parts of the structure [4−6]: this minimum rank is obtained by the knowledge of the modes whose DOFs are most sensitive to the structural modification as indicated evaluating the $\{\varepsilon\}_i$, Eq. 2. In the hypothesis that only stiffness modifications are present, the updated matrix, $[K_u]$, could be obtained by the knowledge of the error matrix, $[\Delta K]$, so that $[K_u]=[K_0]+[\Delta K]$, specifically $[K_0]$ is the initial numerical stiffness matrix, whereas:

$$[\Delta K] = [B]\left([B]^T[\Phi]_x\right)^{-1}[B]^T \tag{3}$$

where $[B]$ being the minimum rank perturbation matrix whose columns are the vectors $[\varepsilon]_i$ defined above. The approach proposed by *Stubbs* [7] forms the vector $\{\varepsilon\}$ evaluating the differences on the eigenfrequencies between the numerical model and the actual experimental structure. The $i-th$ component of such a vector $\{\varepsilon\}$ is given by:

$$\varepsilon_i = \frac{\omega_{xi}^2}{\omega_{ai}^2} - 1 \tag{4}$$

where subscript a stand for analytical values. Imposing an arbitrary variation of a stiffness and/or mass parameter, it is possible to obtain the columns of the sensitivity matrix in terms of variation of the eigen–frequencies of the numerical system with respect to such a parameter:

$$[S_p]_{ij} = \frac{\frac{\omega_i^2}{\omega_{0i}^2} - 1}{\alpha_j} \tag{5}$$

where ω_i and ω_{0i} are the $i-th$ modified, by the parameter α_j, and the reference eigenfrequency respectively. The solution vector, *i.e.*, the set of parameters that quantify the lack of correlation between experimental data and numerical model, is obtained via a least square fitting procedure. The third updating procedure considered in this paper is proposed in [8] which represents the core method used by *FEMTools* commercial code for updating. This is a response–based updating technique, named Predictor Corrector, that uses two correlation functions defined as:

$$\chi_s(\omega_\kappa) = \frac{\left|[H_x(\omega_\kappa)]^H [H_a(\omega_\kappa)]\right|^2}{\left[[H_x(\omega_\kappa)]^H [H_x(\omega_\kappa)]\right]\left[[H_a(\omega_\kappa)]^H [H_a(\omega_\kappa)]\right]} \tag{6}$$

and

$$\chi_a(\omega_\kappa) = \frac{2\left|[H_x(\omega_\kappa)]^H [H_a(\omega_\kappa)]\right|}{\left[[H_x(\omega_\kappa)]^H [H_x(\omega_\kappa)]\right] + \left[[H_a(\omega_\kappa)]^H [H_a(\omega_\kappa)]\right]} \tag{7}$$

The first correlation function, called *shape correlation* is sensitive to differences in the overall deflection shape of the structure, while the second highlights lack of correlation in FRF amplitude. The sensitivity matrix to a variation of the P design parameters, arranged in the vector $\{p\}$ and evaluated for each frequency point, is derived from the previously defined correlation functions:

$$[S_p] = \begin{bmatrix} \frac{\partial \chi_s(\omega_\kappa)}{\partial p_1} & \frac{\partial \chi_s(\omega_\kappa)}{\partial p_2} & \dots & \frac{\partial \chi_s(\omega_\kappa)}{\partial p_P} \\ \frac{\partial \chi_a(\omega_\kappa)}{\partial p_1} & \frac{\partial \chi_a(\omega_\kappa)}{\partial p_2} & \dots & \frac{\partial \chi_a(\omega_\kappa)}{\partial p_P} \end{bmatrix} \tag{8}$$

The resulting sensitivity matrix, of dimension $2N_f \times P$ where N_f is the number of measured frequency points, is derived from derivatives of the dynamic stiffness [8]. The vector $\{\varepsilon\}$ is formed evaluating, again for each frequency point, the two correlation function:

$$\{\varepsilon\} = \begin{Bmatrix} 1-\chi_s(\omega_\kappa) \\ 1-\chi_a(\omega_\kappa) \end{Bmatrix} \tag{9}$$

3 LIMITED DEGREE OF FREEDOM AND NOISE

One of the main problems arising in correlating the numerical model with the experimental counterpart is related to the incompleteness of experimental data. In general, dealing with a representative *F.E.* model, the number of DOFs exceeds the number of measured coordinates, moreover only a limited number of eigenvectors is available from experimental data. This issue could be fronted either expanding the experimental data to the analytical ones [10] or reducing the analytical model to the experimental measures [11, 12]. Although the first approach preserves the physical connections of the whole structure, experience has shown that the latter is generally more successful [9, 10]. In the wide scenario of system reduction or expansion methods, attention is focused on the capability of the so called SEREP approach [10] to deal with incomplete experimental informations: this method has been used both reducing and expanding the numerical system. The key idea of the SEREP method is the introduction of the modal coordinates in the linearized equation of motion yielding to the trasformation matrix:

$$[T] = \begin{bmatrix} [I] \\ [\Phi_{tr}]\,[\Phi_{rr}]^{-1} \end{bmatrix} \qquad (10)$$

where $[\Phi_{rr}]$ is the submatrix of the numerical eigenvector matrix relative to the DOFs retained (subscript r) in the analysis, and $[\Phi_{tr}a]$ is the component eigenvector matrix relative to the truncated DOFs (subscript t). This $N \times r$ trasformation matrix, being N the total number of DOFs in the numerical model, is used to obtain the equivalent reduced system. Considering for example the mass and stiffness matrix, the following relations hold:

$$[M_{rr}] = [T]^T [M] \;\; [T] \qquad\qquad [K_{rr}] = [T]^T [K] \;\; [T] \qquad (11)$$

However, dealing with a reduced set of experimental data, *i.e.*, a reduced number of degrees of freedom, mode shapes, and frequency response functions, with the consequent introduction of the trasformation matrix $[T]$, will result in an ill conditioned sensitivity matrix that could impair the solution of the, in general, over–determined problem presented in Eq. 1. Moreover, it is possible to demonstrate, on the basis of the *SVD* technique for solving linear least square problems, that such a system has unlimited numbers of solutions related to the nullspace of $[S_p]$ [8, 13]. This non–unique parameter estimation shows up, as reported in the following numerical results section, identifying incorrect spatial localization or values of the parameters characterizing the updating procedure. Furthermore, in this paper, experimental data has been simulated by corrupting the numerical FE model at selected elements: noise has been further added in order to introduce the measurement uncertainty. The noise model applied is based on a uniform, zero mean value, random function having the form:

$$\alpha_{noise} = \alpha\,(1 + \ell f_{rnd}) + f_{rnd} b \qquad (12)$$

where the noised, α_{noise}, value is obtained from its corresponding *clean* value, α, and from the uniformly distributed random function f_{rnd}: the noise level, is selected by the coefficient ℓ, $0 \leq \ell \leq 1$, while the overall, α–indipendent bias, is introduced by the value b.

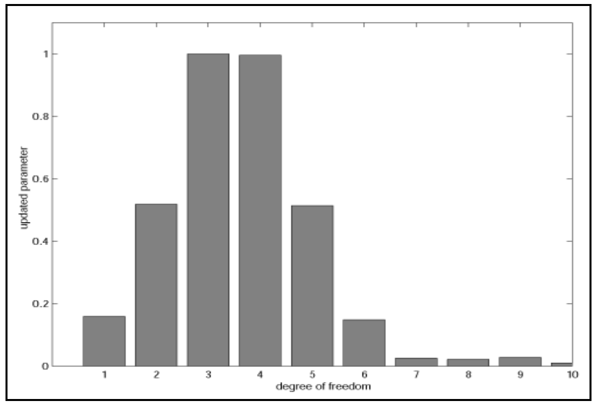

Figure 1: MRP SEREP reduction 4^{th} element modified

4 HYBRID APPROACH

Referring to the *MRP* and the *Stubbs* updating procedures, the following consideration can be made: the first method is sensitive to quite high structural modification with respect to the second one which, in turn, is able to locate, and then update, minimal structural changes. This is due to the different approach they use: as mentioned above the *MRP* method uses the experimental modes as shape functions into which project the numerical system matrices, while the method proposed by *Stubbs* simply involves differences between eigenvalues. Since the latter are most sensitive to structural modifications with respect to mode shape, *i.e.*, the eigenvalue sensitivity is higher than eigenvector sensitivity [14], it is possible to formulate a hybrid updating approach casting together this two approaches. Specifically, an initial evaluation of the mismodelled regions is performed using the *MRP* approach that permits the identifications of regions where relative high differences in the structural parameters could be evaluated, differences that generally are not uniquely identified by the *Stubbs* methodology. These results are used as starting point for the subsequent iterative procedure performing the *Stubbs* approach that could identify the wanted values of the updating parameters. This proposed hybrid approach is intended to overcome the lack of efficiency of the *MRP* and *Stubbs* methods when seeking minimal or high system parameter modifications respectively. The following steps summarize the hybrid approach:

- first evaluation, via *MRP* method, of the mismodelled regions: from the knowledge of the resulting $[\Delta K]$, Eq. 3, the values of ε_i can be termined and can be used as starting point for the subsequent

- *Stubbs* iterative procedure until convergence of structural updating parameters. Final structural parameter modifications can be evaluated solving Eq. 1 where the error vector and the sensibility matrix are evaluated in Eqs. 4, 5 respectively.

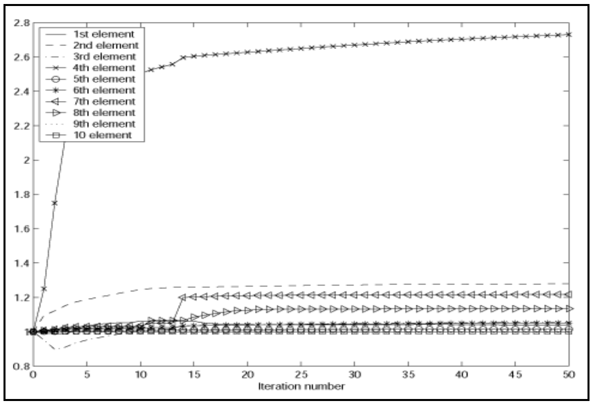

Figure 2: P-C SEREP reduction 4^{th} element modified, step #1

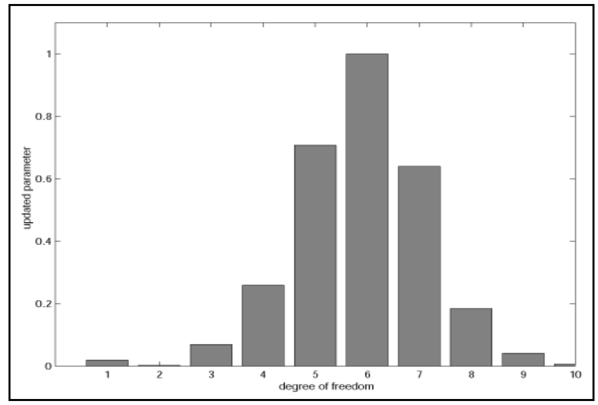

Figure 3: MRP SEREP reduction 6^{th} element modified

5 NUMERICAL RESULTS

The effects of limited DOFs and noise on the three updating methodologies have been investigated on a cantilevered aluminum beam of dimensions $1 \times 0.02 \times 0.01$ $[m]$, with no proper damping, whose dynamic behaviour is predicted by its finite element model formed by ten equally spaced beam elements. The experimental model has been obtained from the analytical one by introducing a stiffness structural modification at selected elements. Approaching the updating problem with a (numerical) complete set of experimental data, *i.e.*, all the degree of freedom, eigenmodes and FRF are present, all the methods permit to correctly identify the modified structural coefficients. First, the effect of the system reduction to the available experimental measurement is presented: the equivalent dynamic characheristics have been obtained simply omitting, from the complete corrupted numerical model, all the quantities assumed not measured. Since the *Stubbs* method is not directly affected by the degrees of freedom used in the updating analysis, *i.e.*, it considers only errors between eigenvalues, only an investigation on *MRP* and *P–C* is presented. The modification factors are applied to the elemental stiffness resulting in a ten component vector $\{\Delta p\}$ to be determined for updating the structure while the degrees of freedom considered in the "experimental" are only the vertical translation at the grid points. Reducing the thickness of the 4^{th} element by 50%, elements are sequentially numbered from 1 to 10 starting from the clamped end, and using the trasformation matrix defined in Eq. 10, the updating parameters are depicted in Figs. 1 and 2 for the *MRP* and the *P–C* technique respectively: the incompleteness of the modal representation has been simulated considering only the first 5 eigenvectors when updating with *MRP* technique. Specifically, Fig. 1 reports the cumulative error vector, [6, 15], adimensionalized with respect to its maximum component value, while in Fig. 2 the convergence history of the modification parameters is presented. The same trends are presented in Figs, 3 and 4 where the mismodelled region is on the 6^{th} element obtained with the same thickness variation. Both methods are not able to uniquely identify the introduced mismodelled parameter: *MRP* technique presents a spatial indetermination that seems to be more important than

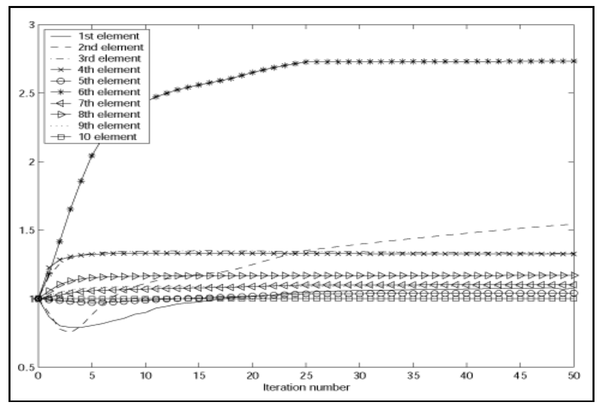

Figure 4: P–C SEREP reduction 6^{th} element modified

the one in *P–C*. In fact, in Fig. 1 the degrees of freeedom interested by the updating procedure range from 2 to 5, *i.e.*, elements from 3 to 5, while from Fig. 3 the interested elements are the 6^{th} and 7^{th}. On the contrary, *P–C* scheme seems to identify the correct parameter with less uncertainty with respect to the *MRP*. From the point of view of the determination

Modif. Elem.	*MRP* error	*P–C* error
IV	-16.0%	-62.5%
VI	-29.5%	-77.5%

TABLE 1: Updating coefficient error (SEREP model reduction)

of the amplitude of the updating coefficients, in Tab. 1 are reported the errors, in percentage, of these coefficients with respect to the correct ones based on the previous analysis and negletting the spatial indeterminations characterizing the *MRP* method: the high level of uncertainty is evident. Similar results can be obtained when expanding the measured data to the numerical model using again the SEREP method to achieve the "experimental" rotational degree of freedom. For the sake of brevity, results from *MRP* are only presented in

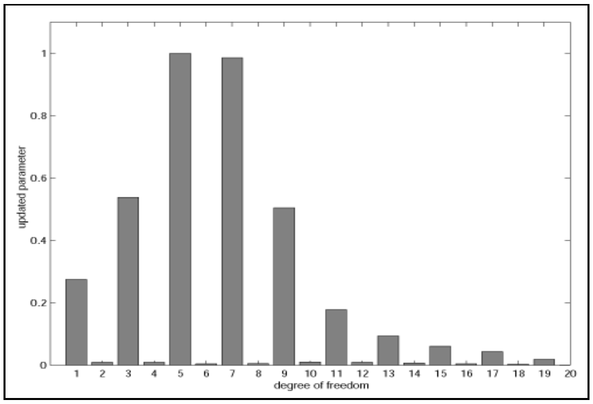

Figure 5: MRP SEREP expansion 4^{th} element modified

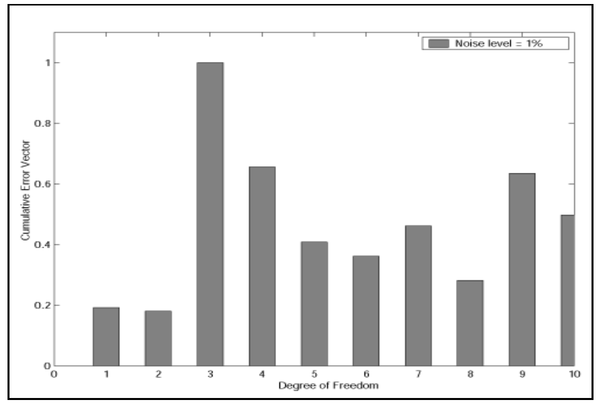

Figure 6: MRP SEREP reduction + noise: 4^{th} element modified

Fig. 5. It is worth noting that in this case no correction prediction for the rotational degree of freedom is identified. The predicted amplitude of the updating parameter results in percentage error less than the reduction case. In Tab. 2 are reported the difference between the predicted and the corrected value of the updating coefficient obtained via *MRP* and *P–C* methods. Keeping the physical connectivity of the analytical model, by expanding the measured data to the numerical model, seems not to introduce as high error in the coefficient extimate as reduction does. Indeed, even using experimental model data expansion, the spatial identification of the mismodelled regions is poor for the *MRP* method, while a good identification, no spatial indetermination and small error, has been achieved, when perturbing the 6^{th} element, by the *P–C* approach. Next, the same cantilever beam has been used to investigate the effect of noise in updating the structural model. Introducing a stiffness modification of -10% in the 4^{th} element of the FE model and corrupting the dynamic characteristics of the equivalent experimental model with a level of 1% of noise and with a low value of bias, Eq. 12, the model updating methods show different behaviours. From Fig. 6 one can see that the spatial identification of the mismodelled region is seriously impaired: most of the degrees of freedom are sensitive to the introduced error. Evaluating the corresponding error stiffness matrix, Eq. 3, an error of 37% on the updating coefficient, with respect to the correct value, has been estimated: the corresponding graphical representation is depicted in Fig. 7 where the in–plane coordinates represents the matrix entries, while the peaks are the corresponding values (in magnitude). The updating coefficients achieved by the *Stubbs* method appear to be more accurate than the one proposed in *MRP* both in space location and in magnitude. In fact, from Tab. 3, not only the correct updating coefficient has been identified, but also its magnitude, with an error of -9.3%: in this Table, for each noise level considered the identified mismodelled coefficients (elements) are reported. It is important to note that this good identification of the *Stubbs* method no longer holds if an high structural modification or noise level is introduced: if a noise level of 5% is considered an incorrect parameter corresponding to the 6^{th} element is identified. Parameters identified by the Predictor–Corrector method, Tab. 3 and Fig. 8, well predict the correct values in term of spatial identification, while their magnitudes are not properly evaluated, errors of -7% and -9% with noise level of 1% and 5% respectively [16]. However, this updating methodology is more robust than the

Modif. Elem.	*MRP* error	*P–C* error
IV	5.4%	-49.6%
VI	-8.2%	-4.5%

TABLE 2: Updating coefficient error (SEREP model expansion)

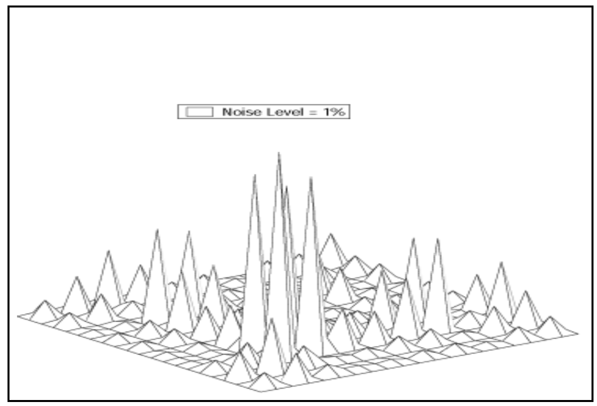

Figure 7: MRP SEREP reduction + noise 4^{th} element modified

Method	Updating Coefficient Error %		
	Noise level = 1%	Noise level = 5%	
	4	4	6
Stubbs	-9.3	-14.3	-0.035
P–C	-7.0	-9.0	—

TABLE 3: Comparison of updating results for Stubbs and P–C methods with noise

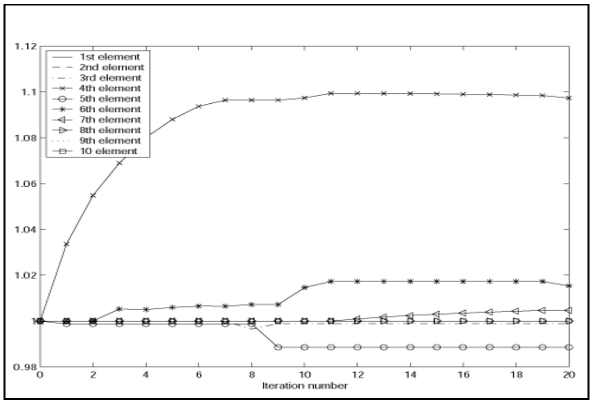

Figure 8: P–C SEREP reduction with noise 4^{th} element modified

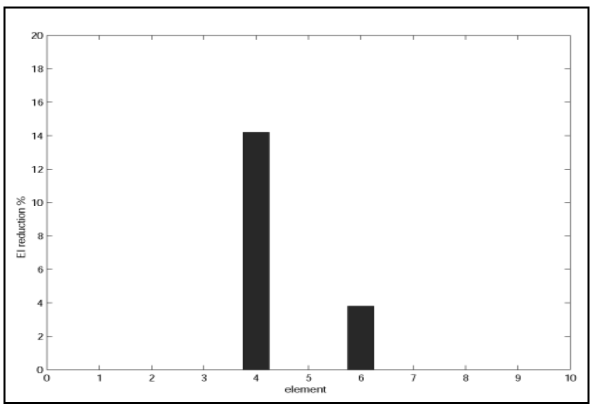

Figure 9: STUBBS SEREP reduction + noise 4^{th} element modified

one proposed by *Stubbs* at higher levels of noise: no wrong elements have been identifiedi. Finally, the proposed hybrid approach is used to update the same cantilever beam. Testing this procedure with different noise levels and different structural modifications, the capabilty to overcome limitations of *Stubbs* and *MRP* method is confirmed. Introducing a noise level of 5% in the "experimental" data assumed to be corrupted at the 4^{th} element with a reduction of its stiffness value by 10%, the *Stubbs* procedure identifies differents updating parameters instead the correct one at 4^{th} element, Fig. 9: as one can see, element 6^{th} is wrongly determined. This

Method	EI reduction	Error
STUBBS	14%	40%
HYBRID	27%	170%

TABLE 4: Stubbs & Hybrid methods comparison

wrong identification seems to be wiped out by using the hybrid approach: Fig. 10 shows that the right spatial coefficient is predicted, while its value it is not yet correctly evaluated. Regarding the magnitude of those coefficients, high errors are

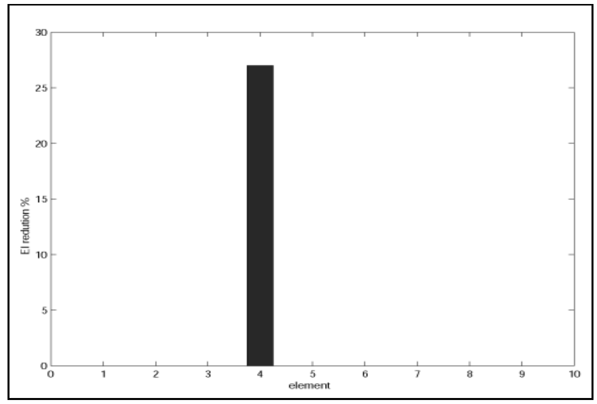

Figure 10: Hybrid approach

still identified as reported in Tab. 4 where the entry regarding the *Stubbs* method refers only to the 4^{th} element, whereas the 6^{th} wrong parameter identification has not been considered: even if the hybrid method shows higher errors than *Stubbs*, this latter approach does not permit a unique identification of the correct parameter.

6 CONCLUDING REMARKS

In this paper the effects of limited DOFs and noise in updating a numerical model of a structural component have been investigated. Among the approaches available in literature, the Minimum Rank Perturbation, Stubbs, and the Predictor Corrector sensitivity based methods have been considered. The limited and noise corrupted set of experimental data has been taken into account by considering only the translational degrees of freedom, and a limited number of eigenmodes, using an uniformly distributed random function. Although these methods properly update the numerical model when using a complete set of experimental data, they present different behaviours when reducing the analytical model to the experimental one, or expanding the latter using the SEREP technique. Specifically, the Minimum Rank Perturbation and Stubbs methods fail to univocally identify the spatial location of the mismodelled regions and this negative characteristic is more evident when dealing with small modifications, for the *MRP* method, or big variations when using *Stubbs*. Introducing an equivalent noise to the experimental data those methods definetly are unable to update the numerical model. An hybrid approach is then proposed to overcome such limitations. From results reported, this mixed method succesfully identify the mismodelled elements, but the value of the updated coefficient still has an high error: this fact suggests an in deep analysis, to be done in future works, regarding the determination of the correct value of the updating coefficient. The Predictor–Corrector method seems to update the structural model in "experimental" conditions. Even if it does not identify the correct magnitude of the updating coefficients, it does not show a significant spatial indetermination as the above mentioned methods. Furthermore, this method has shown a good robustness when introducing noisy experimental data.

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