Article presented at the 23rd International Seminar on Modal Analysis (ISMA), September 16-18 1998, Leuven, Belgium.

Identification of Pressure Forces in a Cavity using an Inverse Solution Method

E. Dascotte, J. Strobbe
Dynamic Design Solutions (DDS), Leuven, Belgium
E-mail: eddy.dascotte@dds.be; http://www.dds.be

Abstract

In many cases, dynamic forces are not directly measurable and need to be identified using inverse methods. A method based on the combined use of experimental responses, serving as reference test data, and finite element analysis is used to identify dynamic forces acting on structures.

An application example demonstrates that measurements of surface velocities, measured at the outside of a car muffler using a laser-scanning device, can be used to identify the pressure forces inside the muffler cavity at any desired excitation frequency. In this case, the pressure forces are assumed to be a linear combination of the acoustic modes of the cavity and the inverse method is used to obtain the participation factor of each acoustic mode. The method requires that, prior to force identification, a finite element model of the muffler has been updated to match its experimental dynamic behaviour in the best possible way.

1. Introduction

Knowledge of the operational loads is required to correctly simulate the dynamic response of mechanical structures under working conditions. Whether the problem is related to strength, durability, or noise, the dynamic characteristics of the structure have to be coupled to the excitation forces to obtain the vibrational or acoustical responses.

Operational loads are due to mechanical or environmental forces (fluids, gas flow, ...). Mechanical forces are often relatively easy to estimate or measure. Environmental forces on the other hand, typically have a stochastical, irregular nature. Furthermore, fluid-structure interactions may make the forces dependent on the structural behaviour. Such operational excitation forces are usually not simply or directly measurable. Although some direct measurement techniques are available (load-cells, strain-gauges, pressure gauges,...), mounting problems related to accessibility or temperature conditions make their application not practical. When analysing a partially closed cavity like a muffler, excited by hot gas flow pressure, such experimental approach is altogether impossible. In the best case, pressure can be gauged at the inlet and outlet of the cavity. Indirect identification of forces or updating of approximate estimates of operating forces are the only practical alternatives.

Indirect identification is based on the solution of a matrix system including all available frequency response functions and vibrational responses [1]. These frequency response functions are characteristic for the structure itself and therefore independent of the actual excitation. Hence, they can be obtained experimentally by means of artificial excitation or analytically from a validated finite element model.

Updating of forces is an iterative procedure that starts from estimated forces and a validated finite element model [2]. With each iteration loop, the estimated forces are adjusted until the residual error between predicted and measured vibrational responses is minimised. Such a procedure is similar to the one commonly used in sensitivity-based, iterative finite element model updating methods [3].

Indirect identification of forces based on an analytical response matrix and iterative force updating both rely on the availability of a valid finite element model. All discrepancies between experimental and numerical responses due to modelling of stiffness, mass and damping can then be excluded and the only parameters are the force locations, directions and amplitudes. However, this requires that the modal behaviour of a FE model and its dynamic responses are first correlated against experimental data. If there is no sufficient correlation, then modelling of stiffness, mass and damping must be updated to the best possible extend. For this purpose, commercial correlation and updating software is available [4].

The number of forces that can be identified using an indirect method is limited by the number of responses. This number should be higher than the number of unknown forces. In addition, the position of the responses should be well divided on the overall structure.

Environmental forces act on the entire surface of the structure surface and cannot always be reduced to some local forces, less than the number of available responses. Alternative approaches are to use force updating, also leading to a least squares solution, but starting from estimated forces, or to define a force model. In general, such a model will be function of only a limited number of variables that can be considered as the unknowns in the identification procedure.

Turbulent gas flow in a cavity exercise distributed, non-uniform, pressure loads on the internal surface of a cavity. Assuming that fluid-structure interaction forces and structural excitation forces can be neglected, the internal pressure forces are described as a function of the acoustic modes of the cavity. The unknown variables of this function are the participation factors of the acoustic modes in the frequency range of interest. For each excitation frequency at which responses are measured, these participation factors can be obtained using an inverse method.

2. Implementation

2.1 Basics of force identification

The proposed inverse method to identify excitation forces is based on the following requirements:

- A finite element (FE) model, capable of representing the dynamic behaviour of the structure in the entire frequency range of interest, is available.
- The modal parameters of the FE model are validated against experimental data.
- Response measurements under operating conditions are available.

In the input-output system theory, the output {X} at n points is a function of the system behaviour [H] and m forces {F} acting on the system.

Indirect force identification results from the relation between force and output:

$$X_{ij} = H_{ij} F_j \qquad (1)$$

where X_{ij} is the response at point i due to a force at point j. The global response level at a particular point is obtained by summing contributions of all ingoing forces:

$$X_i = \sum_{j=1}^{m} H_{ij} F_j \qquad (2)$$

Writing this for all measurement points in matrix form yields,

$$\{X\}_n = [H]_{nm} \{F\}_m \qquad (3)$$

$$\{X\} = [H]\{F\} \qquad (4)$$

With known frequency response functions H_{ij}, identification of the forcing functions is the solution of a complex matrix equation (4).

The frequency response functions are characteristic for the structure itself and therefore independent of the actual excitation. In a pure experimental approach, this property would be used to obtain [H] by means of an artificial excitation.

$$[H] = \frac{\{X^o\}}{\{F^o\}} = \frac{\{X^a\}}{\{F^a\}} \qquad (5)$$

where index o stands for operating conditions and index a for artificial excitation. In a mixed numerical-experimental approach, however, [H] is obtained from the FE model by mathematically synthesising the responses functions from the modal parameters of the structure.

The operational forces can be calculated from solution of equation 4 when dynamic responses (displacements, velocities or accelerations) are measured under operational conditions. These responses must be measured simultaneously because amplitude and phase relation between all vibrational responses must be known.

2.2 Implementation

In order to apply force identification on full-scale industrial models, it has been implemented in the FEMtools software program [4]. This program already includes tools to validate and update a finite element model from modal and FRF measurements. Adding force identification and updating from response measurements therefore is a logical extension. Computation of forces from equation 4 using a mixed analytical-experimental approach requires four steps:

1. Updating an FE model to obtain system mass, stiffness and damping matrices that allow to correctly predict normal modes and FRFs of the structure in a given frequency range.
2. Importing experimental reference responses.
3. Expanding test data in case the experimental responses do not cover all DOF of the FE model,.
4. Computing forces from synthesised FRFs and expanded test data.

There many ways to expand test data. A simple but efficient method is SEREP [5] which is originally designed to expand or reduce modal vectors. In this method, a full (F) displacement vector is expressed as a linear combination of subvectors. These subvectors contain displacement values at the active (A) degrees of freedom common to the finite element model and test.

$$[X_F] = [T][X_A] \tag{6}$$

Transformation matrix [T] is found from the pseudo-inverse of the full analytical matrix and its submatrix:

$$[T] = [X_F]^{+}[X_A] \tag{7}$$

This transformation matrix is then used to expand experimental vectors using equation 6. The advantage of this expansion method is that smoothing of the expanded vectors is implicit and that only displacement vectors are required.

For the last step, consider the equation to compute displacement responses in the frequency range spanning N modal parameters:

$$\{X\} = \sum_{i=1}^{N} \frac{\{\psi_i\}\{\psi_i\}^t\{F\}}{\left(\lambda_{ri}^2 - \omega^2\right)} \tag{8}$$

where λ_{ri}^2 is the i-th eigenvalue of the damped system, ω^2 the excitation frequency and $\{\psi\}$ the normal modes of the structure. Several types of damping can be considered (Rayleigh, modal). Premultiplying left and right side with $\{\psi_i\}^t[M]$ yields,

$$\{\psi_i\}^t[M]\{X\} = \sum_{i=1}^{N} \frac{\left(\{\psi_i\}^t[M]\{\psi_i\}\right)\{\psi_i\}^t\{F\}}{\left(\lambda_{ri}^2 - \omega^2\right)} \tag{9}$$

Because the normal modes are normalised with respect to the system mass matrix, this equation can be rewritten as

$$\{F\} = \sum_{i=1}^{N} \{\psi_i\}^{+}\{\psi_i\}^t[M]\{X\}\left(\lambda_{ri}^2 - \omega^2\right) \tag{10}$$

with $\{\psi\}^{+}$ the pseudo-inverse of the modal matrix.

From equation 10, the forces can be directly computed. In case forces act on only m degrees of freedom, $\{\psi_i\}^{+}$ can be reduced to include only those m degrees of freedom. Because the pseudo-inverse of the modal matrix must be computed, the following remarks apply:

- If the number of independent forces m is higher than the number of modes N, then the system of equations is underdetermined. If initial force estimates are available, a force updating method would be more suitable.
- If the number of independent forces m is less or equal than the number of considered modes N, then the forces can be directly obtained from equation 10 which yields a minimum norm or unique solution.

In the special case where forces are related to acoustic pressure in a cavity, {F} can be approximated as linear combination of M known acoustic modes of the cavity:

$$\{F\} = \sum_{i=1}^{M} a_i \left\{F_i^{acoustic}\right\} \tag{11}$$

The acoustic modes are obtained using using acoustic analysis software. If the number of acoustic modes M is equal to or lower than the number of structural modes N, then the coefficients a_i in equation 11 can be found by combining equations 6, 8 and 11.

3. Application example

3.1 Description

The force identification software is validated on an car muffler, using real-life data. The following considerations were taken into account:

- A valid FE model of the muffler is available. This FE model has been validated and updated using experimental modal and FRF data.
- Forces are considered to be mainly due to acoustic pressure inside the muffler cavity. Other forces are neglected

- To exclude other forces, the inlet of the muffler is excited with white noise [300-800 Hz]. At the same time velocities on the upper and lower surfaces, due to the acoustic excitation inside the muffler cavity, are measured using a laser scanning device.
- It is assumed that the structure is only proportionally damped although it is known that various other damping mechanisms are active.

Force identification is done from the surface velocity measurements, the modal parameters of the FE model and the acoustic modes of the cavity. It is assumed that the internal pressure distribution is a linear combination of the acoustic modes of the cavity and that other forces (fluid-structure interaction, structural excitation at the boundary conditions) can be neglected.

3.2 Results

The procedure to identify forces in this case consists of several steps:

1. The acoustic mesh and modes are imported into the program database (figures 1 and 2).
2. The structural FE mesh and modes are imported into the program database (figures 3 and 4).
3. The acoustic modes are stored as modal displacements at the nodes of the volume mesh modelling the cavity. This model must now be converted into a shell model with pressure data (forces) acting on the surface. This is done by mapping the acoustic mesh on the structural FE mesh and then truncating the acoustic mesh. Because one point in the acoustic mesh is mapped onto several FE nodes, the mapping and truncation involves an averaging of acoustic pressures in neighbouring acoustic elements which is then applied at the paired node. Note that at this point the FE and acoustic mesh are identical but that the acoustic modes are still stored as modal displacements (figure 5).
4. At this time, acoustic pressures are represented using 3 displacement components. The next step is to project each pressure normal to the surface and then convert nodal pressure to nodal force (figure 6). To do this, nodal pressures are divided by the one quarter of the surface of each adjacent element. The acoustical force vector base will serve as the basis for force identification.
5. Laser velocity measurement data for the upper surface of the muffler is imported. The original velocity data being too noisy, a filter is applied that averages the data. Each point is considered to be at the centre of a 3x3 matrix. The filtered value is the sum of the averages of the point with each of his 4 connected neighbours and then divided by four (figures 7 and 8).
6. Next, the scanning grid is mapped on the surface of the muffler. Only those grid locations closest to an FE node are kept. The other grid locations and associated velocities are deleted from the database (figure 9).
7. After expansion of the test data for the nodes at which no test data is available, forces can be identified. The result is presented as participation factors for each of the acoustic modes at each excitation frequency where surface velocities were measured (figure 10) and as colour-coded plots of the force fields (figure 11). From the figures, it can be seen that force fields strongly vary with frequency.
8. Finally, the force vectors are exported in a format than can be used by finite element software for further forced responses analysis.

3.3 Verification

It is in practice impossible to measure the internal pressure forces and compare them with the identified ones. Verification of the result is only possible by re-computing the velocities, using the identified forces, and comparing this result with the measured displacements. This is also verified for the lower surface although this data was not included in the force identification procedure.

The displacement correlation table is shown in figures 12 and 13. A correlation criterion similar to MAC [6] is used. Correlation results for the upper surface range between 70-90% for most of the excitation frequencies. This is less true for the lower surface suggesting that it is required to include test data distributed over the entire surface instead of relying on expansion methods.

The results, however, are considered to be satisfactory considering

- incomplete, noisy test data (expansion and filtering required)
- remaining imperfections of the FE model, even after updating (damping, non-linearities)
- errors due to modal truncation
- no fluid-structure interaction forces or other forces are taken into account

Further visual verification and interpretation of the results can be done by superimposing predicted and measured displacements. (figure 14).

4. Conclusions

A inverse method was presented to identify dynamic forces using a validated FE model of a structure and experimental responses at a number of excitation frequencies. A major manufacturer of mufflers (ECIA) has successfully applied it as part of a project to optimise acoustic performance of its products. The identification software can easily be integrated in an existing analysis environment consisting of various design, simulation and measurement tools.

Further investigations are required to describe and reduce errors due to modal truncation, noisy test data, expansion of test data, and remaining errors in the finite element model. However, it has been demonstrated that by combining laser scanning measurements and an approximate FE model, it is possible to obtain valuable information on unknown internal pressure forces in complex structures like mufflers. The procedure can easily be adapted to work for any type of experimental information (operational deflection shapes, velocity or acceleration) to identify nodal or distributed forces.

Acknowledgements

The authors gratefully acknowledge the support of the EC through BRITE/EURAM project No. BE 7363. The FE model and experimental data was kindly provided by ECIA, France.

References

1. G. Desanghere, R. Snoeys, *Indirect Identification of Excitation Forces*, Mechanical Engineering Department, K.U. Leuven, 1987.
2. E. Dascotte, G. Desanghere, *Updating of Force Functions*, Proceedings of the 9th International Modal Analysis Conference (IMAC), April 1991, Florence, Italy.
3. J. Collins, G. Hart, T. Hasselman, B. Kennedy, *Statistical Identification of Structures*, AIAA journal, 12/2, February 1974, pp. 185-190.
4. *FEMtools User's Guide*, Version 1.4, Dynamic Design Solutions, June 1998.
5. J. O'Callahan, P. Avitabile, R. Riemer, *System Equivalent Reduction Expansion Process (SEREP)*, 7th IMAC, 1989.
6. R. Allemang, D. Brown, A Correlation Coefficient for Modal Vector Analysis, 1st IMAC, 1982.

Figures

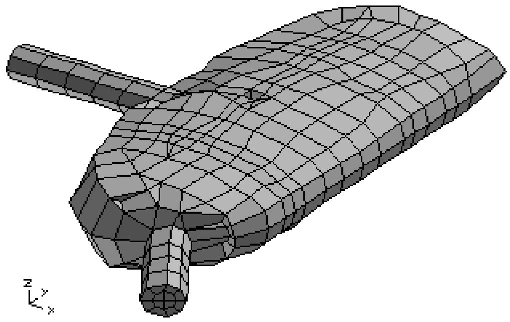

Figure 1: Finite element model of a muffler cavity used for acoustical modal analysis using 3D volume elements.

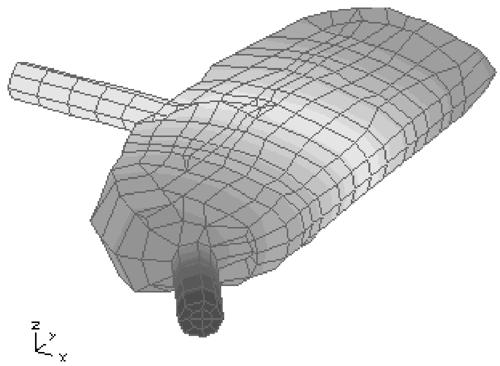

Figure 2: Example of an acoustic mode (256 Hz; modulus).

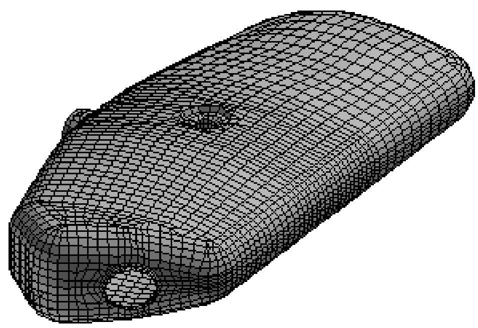

Figure 3: Structural finite element model using shell elements.

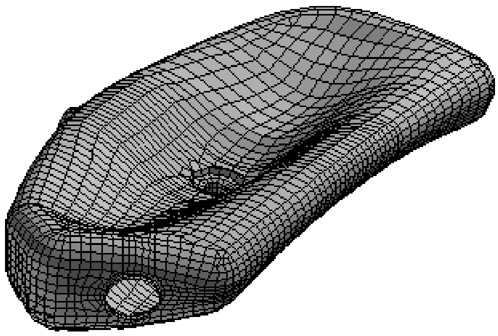

Figure 4: Example of a structural mode (260 Hz

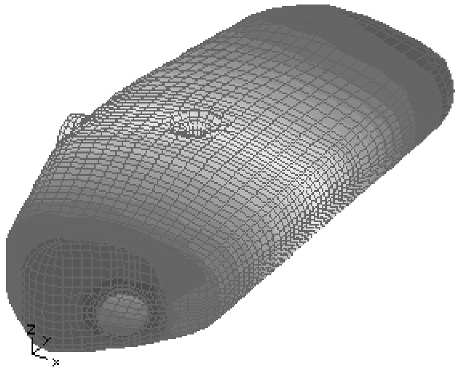

Figure 5: Acoustic mode after mapping on FE mesh (256 Hz).

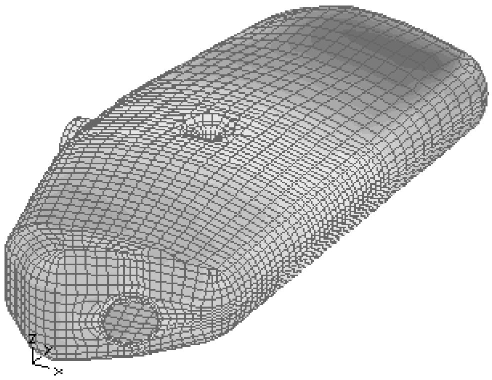

Figure 6: Nodal force distribution obtained from the acoustic mode at 256 Hz.

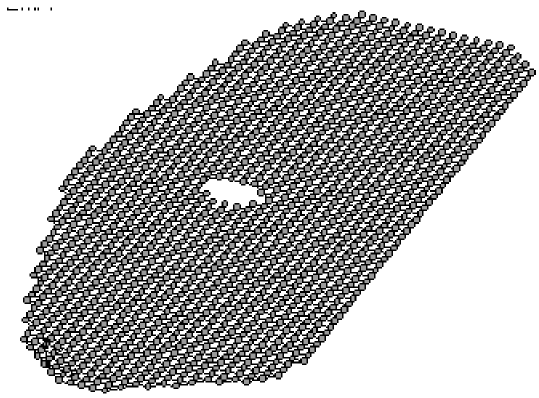

Figure 7: Scanning grid for laser velocity measurements.

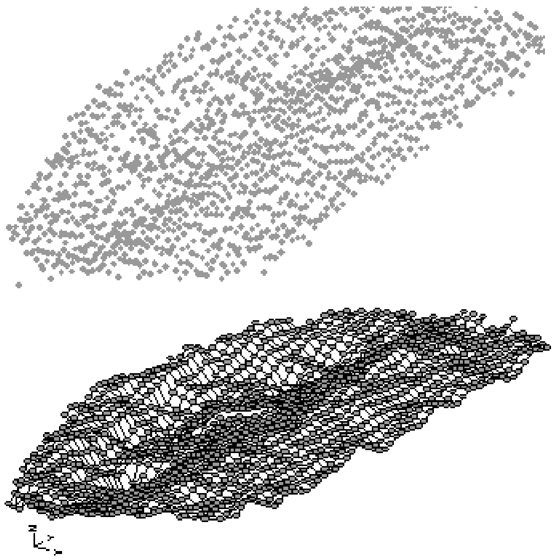

Figure 8: Laser velocity measurements at 386.2 Hz (top = unfiltered, bottom = filtered).

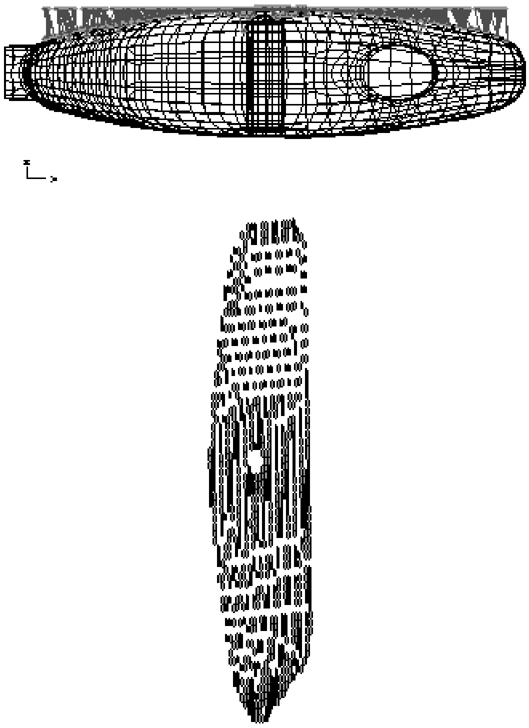

Figure 9: Mapping the scanning grid onto the muffler surface (top = mapping; bottom = reduced scanning grid)..

```
DYNAMIC EXCITATION FORCE IDENTIFICATION
EXCITATION          :          1
PARTICIPATION FACTORS
      VECT           REAL      IMAGINARY
         1 -0.404800E-01  0.109077
         2  0.885560E-01 -0.188120
         3 -0.161135E-01 -0.139174
         4 -0.237507      0.542591
         5  0.596707E-01 -0.104541
         6 -0.258851E-02  0.295065E-01
         7 -0.235892E-01  0.423879E-01
         8 -0.944698E-02  0.199800E-01
         9 -0.166131E-01  0.599786E-01
        10  0.109827E-01 -0.433330E-01
        11  0.502019E-02 -0.394135E-01
        12  0.155390E-01 -0.235219E-01
        13 -0.137111E-01  0.307866E-01
        14 -0.132813E-01  0.413881E-01
        15  0.729303E-02 -0.247328E-01
        16  0.428704E-02 -0.110684E-01
        17 -0.626452E-02  0.169691E-01
        18 -0.371021E-03  0.990164E-03
        19  0.519295E-03 -0.137248E-02
        20  0.601062E-03 -0.363239E-02
```

Figure 10: Identified participation factors for the first excitation frequency.

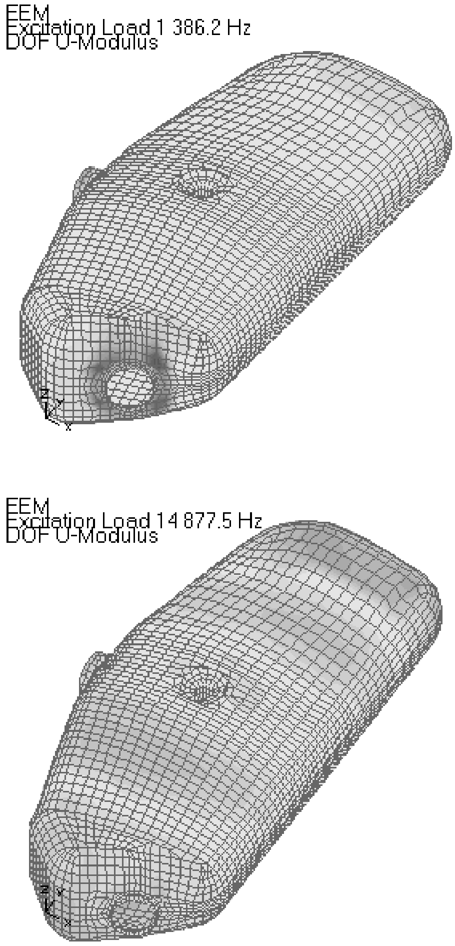

Figure 11: Visual representation of the force distribution at two different excitation frequencies (386.2 Hz and 877.5 Hz).

FEA	TEST	DAC Upper Surface (%)	DAC Lower Surface (%)
1	1	84.5	-
2	2	71.2	-
3	3	84.7	20.7
4	4	76.7	26.1
5	5	18.2	-
6	6	61.3	-
7	7	92.0	45.2
8	8	87.0	21.0
9	9	88.5	24.6
10	10	72.8	32.1
11	11	88.4	66.9
12	12	49.5	16.9
13	13	25.8	28.7
14	14	73.5	-
15	15	20.8	12.1
16	16	7.3	-
17	17	70.0	-

Figure 12: Correlation in terms of Displacement Correlation Criterion (DAC) between measured and computed displacements at 17 excitation frequencies.

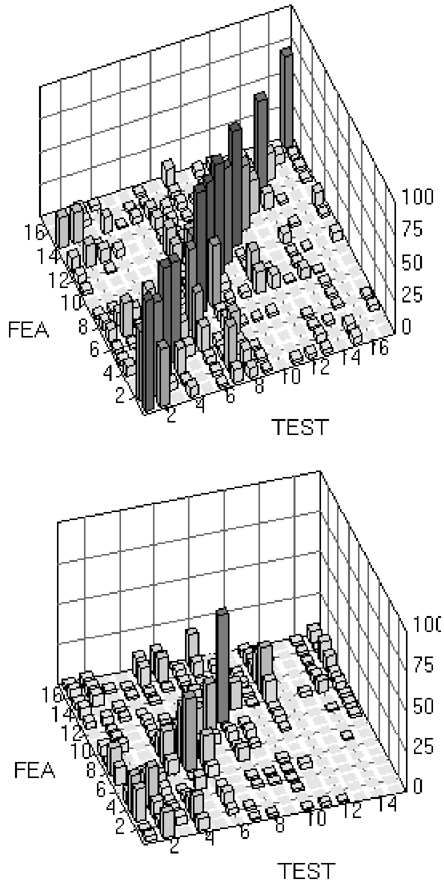

Figure 13: Displacement Assurance Criterion matrix showing correlation between predicted and measured displacements at 17 excitation frequencies (top = upper surface; bottom = lower surface).

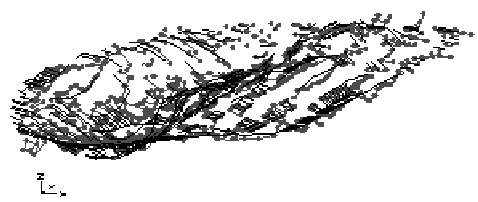

Figure 14: Superposed pair of predicted and measured displacements at excitation frequency 386.2 Hz, showing a FE mesh reduced to the size of the scanning grid.