Summary:

On the firefighting boom of a MI 17 helicopter an experimental modal analysis has been carried out in four different configurations. The results revealed significant differences to the analytical data. A resonant excitation by the downwash forces of the propellers of the second vertical bending mode of the boom was detected. In order to predict the excitation of the boom in flight the analytical model was updated. All four configurations were modeled. Since parameter variation affected all four models a so-called “multi-model updating” technique was used. After additional flight tests without boom and ground tests with boom it was possible to assess the relevance of the resonant situation. In the final operation deflection shape test in flight with boom the assumptions were verified. Finally the acting dynamic downwash forces were updated using the ODS data. Though the structure is not very complex the analytical model revealed significant errors leading to a hazardous mis-assessment. Thus the boom serves as a striking example how methodic model updating based on various test data is necessary to produce validated models and reliable assessment.

Keywords:

Multi-Model Updating MMU, Force Identification, Experimental Modal Analysis, Operation Deflection Shape Analysis ODS.
1 Introduction

Kuala Lumpur is one of the vastly growing Asiatic cities with high rising buildings, among them the tallest buildings in the world, the Petronas towers. With the development of the tall edifices also the risk of local fire in upper floors increases which might be difficult to access rapidly by fire fighting personal. To enhance a first and quick response at a critical stage of the fire the firefighting department BOMBA of Kuala Lumpur is equipped with firefighting helicopters. Comparable helicopters are in service in the US and in Japan. The latter are equipped with a boom in flight direction to shoot water.

The concept developed by AIRROD Malaysia however provides a lateral cantilever boom mounted perpendicular to the helicopter fuselage. This allows better maneuvering in case of sudden critical incidents like explosions. The boom itself consists of an aluminum girder with three pipe shaped main bars that are stiffened by x-braces (fig. 1). The lower pipe is simultaneously used as a water pipe with a nozzle at the tip. The boom itself can be driven by a hydraulic device to move it to a parallel position along the helicopter fuselage. In this position another attachment is provided next to the pilots compartment. The clamp can be operated by the pilot.

Figure 1: Top view of the boom in operating (perpendicular) and maneuvering (parallel) position, replacement of the rigid boundary conditions by stiff spring elements.

Preliminary numerical analysis have been performed to investigate the dynamic behavior of the boom [1] in four different configurations:
- boom in operating position (perpendicular), empty.
- boom in operating position (perpendicular), pipe filled with water.
- boom in maneuvering position (parallel, clamped), empty.
- boom in maneuvering position (parallel, clamped), pipe filled with water.

The mass of the water in the pipes increases the total weight of the boom (m = 57 kg) by 40 %. the dynamic behavior is significantly changed.

The safety of the boom has been verified for the following load cases:
- Static load of the inertial forces of the boom itself.
- Static load by downwash during hovering.
- Static load by wind on maneuvers.
- Dynamic load by downwash during hovering.

The assessments of the last load case were mainly based on the prediction of the modes of the boom. The main dynamic forces are the downwash forces excited by the rotor blades at 16.2 Hz (rotation speed: 196 UPM x 5 rotor blades). No critical eigenfrequency was found in the vicinity of this excitation frequency and therefore a resonant situation can be avoided.

The task of the following investigations by Mueller-BBM initially was to validate the analytical results by an experimental modal analysis (EMA) and an operation deflection shape analysis (ODS).

2 Modal analysis (EMA)

The experimental modal tests were performed for the four different configurations. In the parallel position also the situation was considered when the boom is not yet attached to the clamp. The boom was excited at the tip with an impact hammer. At up to 8 testing points the acceleration response was measured simultaneously at a maximum of 15 channels in vertical and horizontal direction (fig 2a). For data acquisition a multi-channel VXI hardware in connection with the PAK data acquisition software was used [7].
Exploiting the response data the parameter identification revealed mode shapes and eigenfrequency in the range from $f = 16.2$ Hz (parallel with water) up to $21$ Hz (perpendicular w/o. water) for the second vertical bending mode of the boom (fig. 2b). Thus the fundamental assumption of the analytical investigation that no resonance occurs near the excitation frequency of the downwash forces was obsolete.

3 Model-updating

3.1 Substitution of the boundary conditions

The reason for this discrepancy of the original analytical results and the testing has been located in the stiffness of the brackets (fig 3). By lack of data the original analytical model assumed the attachments to the helicopter to be rigid. As was shown by the experiment this assumption was approximately true for the operating condition but not at all for parallel position. To assess the situation a valid model was required.

With the boom detached the horizontal and vertical impedances at the brackets on the helicopter fuselage were determined experimentally. The lower impedances which are more significant for the eigenfrequencies revealed approximately stiffness characteristics. Therefore it was appropriate to replace the rigid boundary conditions by elastic stiffness (fig. 1). Hence a much better correlation was achieved, but still better accordance was desired especially near resonance. Instead of detailed modeling of all the fuselage of the helicopter updating of the boundary stiffnesses seemed to be the more appropriate way to attain satisfactory correlation.

3.2 General approach

For further tuning of the model the widely used sensitivity based updating technique as described for instance in [2] was used. The main steps shall briefly be summarized. The error in the response values e.g. eigenfrequencies were introduced as residuals of a non-linear optimization algorithm. The standard approach adopted in model updating is to linearise the estimation problem about the current parameter estimate, and to iterate until convergence.

Relating the necessary parameter update $\Delta P$ with the error in the response $\Delta R$ a gradient sensitivity matrix $S$ is computed by (1).

$$[S] = S_{ij} = \left[ \frac{\partial R}{\partial P_j} \right]$$

(1)

For a series of parameter-response combinations closed forms are given so that expensive system solving with varying parameters is avoided. Eq. 2 represents the sensitivity of an eigenfrequency.
To solve the equation for the parameter update $\Delta P$ the sensitivity matrix must be inversed. Since the number of parameters rarely corresponds to the number of responses the sensitivity matrix is not quadratic. For the inversion a least square method is used to create a so called pseudo inverse. However there are a number of issues relating to the conditioning of the estimation problem that must be addressed. To overcome these problems weighting is applied to both the measured data and the parameters with the following advantages:

- An ill-conditioned inverse sensitivity matrix caused by certain parameter - response combinations can be manipulated.
- Thus different classes of responses, e.g. eigenfrequencies and MAC-values can be treated simultaneously.
- The optimization algorithm can be regarded as a Bayesian parameter estimation algorithm where the weighting matrices are related to known or estimated standard deviations of the parameters and responses.

Since the $\Delta R$ is not differential and the relation between parameters and responses is non-linear the optimization has to be performed iteratively. A new response with the updated database is computed and compared to reference, then the next parameter update is computed based on a new sensitivity matrix. The loop is repeated until satisfactory convergence is achieved.

3.3 Multi-model updating

In difference to the classical updating procedure where the responses of one model are used as residual functions here the results with four different configurations were subject of the updating. The fact had to be considered that a parameter change causes a different response in all four analytical models. Therefore the method had to be extended to a so called multi-model-updating technique [3]. A global sensitivity matrix (and weighting matrix) is assembled of submatrices descending from the individual models. As the parameters are the same in all four models the number of rows of the sensitivity matrix is fixed whereas the number of columns is increased by the different responses of each model.

In the present investigations the error of the eigenfrequencies of the 2nd, 3rd and the 4th mode were defined as residual functions. The 4th eigenmode was weighted higher in order to force the algorithm to focus on the reduction of the error at the second vertical bending model. The boundary stiffness of the brackets $k_x (2)$, $k_y (2)$ and $k_z (1)$ as well as the longitudinal stiffness of the hydraulic device $k_{II}(2)$ were defined as parameters. Scaled with higher confidence also global parameters like Young’s modulus “E”, the density of the material “rho”, the pipe diameter “Diam”, the nozzle mass “m” and the mass of the water “H20” were chosen. Figure 4 shows the complete sensitivity matrix.

It should be noted that off course the mass of the water was only considered in the two respective models simultaneously as indicated in fig. 4. The longitudinal stiffness of the hydraulic device were allowed to be different in the perpendicular and parallel position and thus required separate treatment.

![Figure 4: Multi-model sensitivity matrix](image-url)
Before discussing the results it should be noted that the success of the updating depends on the choice of the parameters and responses. Furthermore weighting plays an important role. By consequence there is no unique solution to the problem. However this should not be considered as a drawback of the procedure but rather as an opportunity for the engineer to explore the analytical model and its performance to show certain phenomena.

In this example a sequence of combinations has been examined to enhance the correlation without loosing too much of the physical background of the structure. For this purpose a software tool is required where the possibility of simultaneous examination and correlation of a test database and an analytical database is provided. A powerful graphical user interface is required to visualize the complex arithmetic.

The core of the described model updating procedure is the computing of the sensitivities. The software must therefore supply a library of closed forms for all parameter-response combinations of the database.

Finally the database and the standard routines like sensitivity computation must be for manipulation. Each problem requires its customized routines. For the multi-model updating it was necessary that the standard procedure was customized to access more than one project file within the algorithm.

With FEMtools [8], specialized software for integration of test and analysis was used that fulfills these requirements.

3.4 Updating results

The updating results are summarized in figure 4. Starting from an error average in the 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} eigenfrequency of more then 50 \% of the original model the error was reduced to 15 \% simply by the introduction of stiff boundaries instead of rigid ones. By updating the error was further decreased to less then 5 \% for all 4 models or less then 1 Hz in absolute values. For the most critical configuration in parallel position with water the error is less than 1 \% at 16.2.

![Figure 5: Error of eigenfrequency before and after updating using modal response functions.](image)

As a side effect also the correlation between the experimental and analytical shape functions was improved significantly. In figure 6 the MAC matrices for the corresponding models is depicted. The matrix pairs each analytical Eigenvector $\Psi_a$ with each experimental eigenvector $\Psi_e$. A high value signifies good correlation.
Initial model, fixed boundaries

Initial model, stiff boundaries

updated models: perpend. / perpend. water / parallel / parallel water

Figure 6: MAC-matrices: Correlation between experimental and analysis eigenvectors, comparison between initial and updated models.

Figure 7: Mode shape pair at 16.2 Hz (parallel, water).

Fig. 7 shows the mode shape pairs of the 2nd vertical bending mode in parallel position with water at \( f = 16.2 \text{ Hz} \).

Generally the model would now be fit enough for further analytical investigations like e.g. stress analysis. Since the prediction of the response in flight condition was to be compared with Operation Deflection Shape testing (ODS) the model was further tuned to match also the Frequency Response Functions (FRF) of the experimental modal analysis [4].

When defining all existing measured FRF functions as residuals, one realizes that a lot of constraints have to be regarded. In addition the sensitivities vary enormously near resonance. Therefore the Cross-Signature-Assurance Criterion (CSAC) is introduced as an error function (fig. 8).

Figure 8: frequency spectrum of the CSAC-criterion (continuous line): correlation between experimental and analytical FRF-data.
The CSAC is computed in analogy to the MAC for each paired FRF in each discrete frequency step. A high value between 0 and 1 indicates good correlation between a series of paired FRF functions. The CSAC is less sensitive in the vicinity of resonance peaks.

As a result it was also possible to match the FRF in a satisfactory way (figs. 9) especially around the critical frequency range of \( f = 16.2 \) Hz.

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\text{Figures 9: FRF-pairs, analytical (continuous) and test (stepped).}
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### 4 Flight tests, Operation Deflection shapes

After several investigation steps including stress analysis, ground test, flight test without boom it was possible to predict that the resonant situation in the second bending mode was not critical compared with other load cases. One reason was that the 2nd bending mode of a cantilever beam is antimetric whereas the downwash forces can be assumed to be uniformly distributed. Nevertheless it was possible to show that due to the resonant excitation the response of the boom was significantly higher in parallel position with water. Over a time window of 1200 s the acceleration response of the boom at 16.5 Hz as well as at 19.25 Hz (resonance in operating position) is traced in fig. 10. Apparently the amplitudes are increased by a factor of 5 to 6. Figure 10b depicts the overlayed APS spectra of acceleration at \( t = 816 \) s. The peak at \( f = 16.2 \) Hz is clearly visible.

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\text{Figure 10a: Time tracks of the amplitudes at } f = 16.5 \text{ Hz and } 19.125 \text{ Hz at the tip of the boom.}
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\text{Figure 10b APS of acceleration at } t = 816 \text{ s.}
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### 5 Force Identification

For further analysis on modified structures it was desirable to have a validated load model. Since the downwash forces were difficult to measure, indirect identification of the forces or updating of approximate estimates of operating forces was a practical alternative. This procedure requires a validated finite element model [5-6]. The i original estimated load vectors \( F_{e,i} \) are scaled by an unknown factor \( \alpha_i \) according to eq. 3.

\[
\left\{ \alpha_i \right\} = \left\{ \sum_{j=1}^{N} \alpha_i \right\}.
\]

To determine the scaling factor the equation to compute displacement response vector \( \{X\} \) in the frequency range spanning \( N \) modal parameters is inverted:

\[
\left\{ X \right\} = \left( \sum_{j=1}^{N} \Psi_j \right)^{-1} \left\{ \left[ \psi_j \right] \left[ \psi_j \right]^T \right\} \left\{ F_{e,j} \right\}
\]
λ₀² is the i\textsuperscript{th} eigenvalue of the damped system, \( ω² \) the excitation frequency and \( \{Ψ\} \) are the normal modes of the structure. Since the number of DOF of the experimentally determined displacement vector \( \{X\} \) is much smaller then the number of DOF of the analytical model, a system reduction method like Guyan reduction or the System Equivalent Reduction Expansion Method SEREP precedes.

Figure 11: Identified vertical dynamic forces at \( f = 16.2 \, \text{Hz} \).

The result of the force identification is shown in fig. 11 for the vertical direction. Applying the updated force vector in equation 4 again an analytical displacement vector can be compared to the test results of the ODS. In analogy to the MAC a Displacement Assurance Criterion (DAC) is computed to get a quantitative correlation indicator. For the paired displacement shapes in fig. 12 a correlation factor of DAC = 75 \% was attained. Considering the only approximate stationary condition this is an acceptable result.

Figure 12: ODS at \( t = 816 \, \text{s} \) and \( f = 16.2 \, \text{Hz} \) (experimental).

6 Conclusions
- The investigation of the helicopter boom served as an example to show the variety and the efficiency of model updating techniques.
- The feedback from the customer and even the pilot proved that throughout the validation of the model a high confidence could be established in this safety-relevant concern.
- The fact that during the flight tests the responsible engineer had to be on board gave the demand of quality management and validation also a very personal aspect.
- Regarding the tight time constraints it was indispensable to work with professional and validated specialized software that assures a reliable and adaptable environment.
7 References


8 Software Used